1. CALCULUS

(1) Question 1.
(a) Calculate \( \frac{d}{dx} \left( cx^c + c^2 x^{-c} \right) \), where \( c \) is a non-zero constant.
(b) Determine \( y' \) where \( y = \frac{1}{\sqrt{2 - 3x}} \).
(c) Let \( f(r) = (r^3 + 1)(-5 - r^-4) \). Calculate \( f'(r) \).

(2) This problem deals with the following function:
\[
f(x) = \frac{4(x + 1)(x - 2)}{(x - 2)(x - 3)^2}
\]
- Calculate all right-hand and left-hand limits of \( f(x) \) at its vertical asymptotes.
- Does this function have a horizontal asymptote? Verify and explain.

(3) Given \( \epsilon = 0.1 \), determine \( \delta > 0 \) such that
\[
|G(x) - 7| < \epsilon \text{ whenever } 0 < |x - 3| < \delta.
\]

2. COMBINATORICS

Let \( n \) be a positive integer. A lecture hall partition of length \( n \) is a partition \( \lambda = (\lambda_n, \ldots, \lambda_2, \lambda_1) \) (where one or more \( \lambda_i \) may be zero) such that
\[
0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n.
\]

Given \( w = (w_1, w_2, \ldots, w_n) \in \tilde{C}_n/C_n \), create the partition \( \lambda = (\lambda_n, \ldots, \lambda_2, \lambda_1) \) with
\[
\lambda_j = \sum_{i=1}^{j} I_{i,j}.
\]
This construction is a bijection between minimal length coset representatives of \( \tilde{C}_n/C_n \) and lecture hall partitions of length \( n \).

The runners corresponding to \( i = 1, 2, \) and \( 3 \) are runners 6, 5, and 3; \( \lambda_{R(1)} = \lambda_1 = 12, \lambda_{R(2)} = \lambda_2 = 12, \lambda_{R(3)} = \lambda_5 = 7, \lambda_{r(1)} = \lambda_6 = 5, \lambda_{r(2)} = \lambda_7 = 5, \lambda_{r(3)} = \lambda_5 = 7, \) and \( \sigma_\lambda = (3, 1, 1) \). Therefore, \( l(W(\sigma_\lambda)) = 3 + (12 - 5) + (12 - 5) + (7 - 7) + 0 \cdot 4 = 17. \)