Chris Hanusa Nov 14, 2003

Math 381 – Homework 5 **Answer Key**

Question 1 – Try to find a graph (on a small number of vertices at least 6, not more, than 15), which shows the traffic-situation of the Seattle-area, and try to show "bottlenecks" on your model.

Answer [10 points] – If you visit
`http://www.cityofseattle.net/html/traffic.htm` you can find the a map of the Seattle Traffic System. This is a good starting point; create a graph starting in Lynnwood south to SeaTac which has a vertex for every intersection and an edge for roads that connect vertices. Then give each edge a capacity, based on the amount of lanes or the volume of traffic that the corresponding road can support.

A bottleneck in this model is what happens when the flow exceeds the capacity. For example, the flow input in the morning from SR-520 east and I-5 south onto the bridge in the University District exceeds the flow output onto I-5 south into Seattle, giving us a traffic jam, i.e. a bottleneck.

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Question 2 – Construct a network connecting the nodes s and t. Let the edges be oriented, and give capacities to them (at least 5 different numbers should occur among the capacities). Let the number of nodes (vertices) in your graph be at least 7, together with s and t. Let it contain at least 3 disjoint (different) paths connecting s to t. It should contain at least 3 cycles not going through s and t. Find a max flow in your network, and prove the maximality of it.

Answer: Your graph got 1 point if you had a network drawn, 1 point if the edges were oriented, one point if you had 5 different capacities, one point if it had more than 6 vertices, one point if you had three disjoint paths, and one point if I could find three cycles. Then four points were distributed on how well your found a max flow and explained why it was max flow. Normal answers had to do with the max-flow/min-cut theorem or finding no paths in the auxilliary graph.

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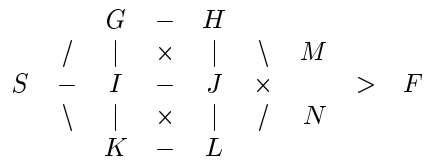
Question 3 – Find a max flow and min cut of graph 1



Answer [10 points] – Sorry, I don't have a copy of the graph. See problem 4 for the type of answer I was expecting. In this problem, the max flow was 14, and the min cut was also 14, with the nodes in the cut $\{S, G, H, I\}$. •

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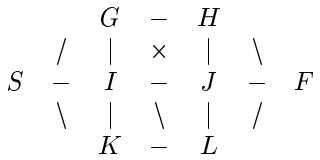
Question 4 – Find a max flow and min cut of graph 2



Answer [10 points] – (a) First we notice three nice flows from $S \to F$, namely $(1) : S-G-J-F$ of flow 3, $(2) : S-I-H-F$ of flow 3, and $(3) : S-K-L-F$ of flow 4. So we have filled the demand for 10 units by inspection and we hope to augment this flow. Reversing the arrows in the limiting paths of the flows above (i.e. $S-G$, $H-F$, and $L-F$), we now check to see if there are any more paths that remain from $S-F$.

There are three augmenting paths. $(4) : S-K-I-H-J-F$ of flow 1, $(5) : S-I-J-F$ of flow 1, and $(6) : S-I-L-F$ of flow 2. After reversing the limiting arrows, we notice there are no more paths from $S-F$, so we have acheived a maximum flow of 14.

To prove that this is a maximum flow, we need to find a minimum cut also of flow 14. So what that means is we need to partition the vertices into two sets A and B where $S \in A$, $F \in B$, and the flow from A to B is 14, i.e. $\delta_A^{\text{IN}} = 14$. We can do this as follows:

Let $A = \{S, K, I, H\}$ and $B = \{G, L, J, F\}$. Then the flow from A to B are the edges $S-G$ of flow 3, $K-L$ of flow 4, $I-L$ of flow 2, $I-J$ of flow 1, $H-F$ of flow 3, and $H-J$ of flow 1. So $\delta_A^{\text{IN}} = 14$, so the partition A, B is a min cut of flow 14, establishing the maximality of the flow of Graph 1 to be 14. •