Chris Hanusa Math 381 – Homework 1 **Answer Key** October 1, 2003

Question 1 – Three officers - a president, a treasurer, and a secretary – are to be chosen among four people: Ann, Bob, Cyd, and Dan. Suppose that for various reasons, Ann cannot be president and either Cyd or Dan must be secretary. Drawing a decision-tree figure out how many ways can be the officers chosen?

Answer – In the following decision tree, the first branching is where we decide who is secretary. There are two choices, Cyd and Dan. Then we choose who is president. There are two choices, Bob and Cyd or Dan. Then we choose the treasurer from the remaining two choices.

`[Insert tree here. Sorry, I don't know how to do them in TEX.]`

As these are the only possibilities, there are 8 possible assignments.

(Worked with Eric B. and Tracey M.)

`[Always make sure to indicate who you worked with, if anyone!]`

Question 2 – Look at the part of a map from the Rand McNally Road Atlas which shows distances and estimated driving times between various cities in the west.

a) Suppose you want to drive from San Francisco to Denver. What is the best route to take? (First address the meaning of 'best'.)

b) Suppose you are planning a trip for next summer and would like to visit a number of places: Yosemite Village, Old Faithful, Denver, Salt Lake City, San Francisco, and Flagstaff. Your trip will begin and end in Seattle. What is the best route to take? You do not need to solve this problem, but explain what would be involved in doing so. Are there other constraints that you might want consider before trying to answer the question?

Answer [10 points] – (a) There are many possible "best" routes that one might consider. Because I like trips that take a short amount of time, we will let "best" mean the shortest time trip.

Considering only paths starting in San Francisco, ending in Denver, heading mainly East that are south of Idaho and north of Arizona, we add the times for each path and find that the smallest amount of time that we can take is the path that visits SF → Sacramento → Reno → Winnemucca → Wells → SLC → Rawlins → Cheyenne → Denver. The optimal value is 18 hours and 56 minutes.

[`It was important that you noted what you designated as the "best" route. Most people did not say what method they used to calculate their route. All I was looking for was a mention of brute force or an algorithm. For your reference, the shortest distance was 1222 miles and the shortest time was 18 hours and 56 minutes`]

Answer [10 points] – (b) Again, we need to consider what the best route is. Then we need to figure out the constraints on this trip are. For example, let's say we have to go to Denver before Flagstaff because Jill's uncle in Flagstaff wants a souvenir from Denver. Maybe we're going at a time when mountain passes are not open. Maybe we want to get to San Francisco for the yearly festival on August 7th. Maybe our car is only going to make it 5000 miles and then die. Maybe we have to get back to school before September, and we only have a certain amount of money to spend on this once-in-a-lifetime road trip. These are all constraints.

To solve this problem, we would need to take into account these constraints, and then compute milage between all the cities, set up a directed graph, and find a Hamiltonian path starting and ending in Seattle that minimizes cost, perhaps.

[`There are many other valid approaches. A full-credit answer must address the constraints and give a valid approach. ALWAYS MAKE SURE TO ANSWER THE QUESTION ASKED.`]

Question 3 – Suppose we are trying to design a ferry schedule for a set of islands, similar to the San Juan Islands in the Puget sound. There are 8 islands with different personalities. Some are big tourist centers, others are "bedroom communities", with many people commuting to mainland jobs each day, and others are still fairly rural. What are some of the issues you would need to address in designing a ferry schedule for this set of islands? Write down some questions you would need to answer before starting, and some ideas as to how you might go about modeling this and what sort of strategies you might consider in designing a schedule. Can you identify any mathematical structures that would naturally arise?

Answer [10 points] – Some issues: We will have to deal with commuters, commerce, and vacationers. Commuters will want a ferry in the morning to the central hub, and away in the evening. Vacationers will increase the need for ferries in the summer months and on weekends.
Some questions: Where do people want to go? When do they want to go?
What is the population of the each island? Do they need to transport cars?
How long does it take to get from one island to the next?
How much does a ferry cost? How many passengers can it hold?
How much money do we have in our budget? Who do we need to satisfy first?
Mathematical structures: The most apparent (to me) is a (directed) graph on 8 (or 9) vertices, with nodes for each of the destinations and edges for possible ferry routes.
Some modeling ideas: Use a graph to do a Network flow problem / Traveling Salesman Problem or use a markov chain to do Stochastic Methods (then say how to apply these)

[`A full-credit answer must address all the issues involved. Again, always make sure to answer the question.`]

Question 4 – We throw two dice. What is the expectation of

a) the sum of the numbers,

b) the product of the numbers?

Show your work.

Answer [10 points total] – Create a table of possible outcomes, with each of 36 possible values $\{v_i\}$ having probability $p_i = 1/36$. Then take

$$E\{V\} = \sum_{i=1}^{36} v_i p_i.$$

(a) $\sum_{i=1}^{36} v_i p_i = 7$

(b) $\sum_{i=1}^{36} v_i p_i = 12.25$

[`That was the hard way. The easy way is to note that rolling two dice are independent events:`]

$E\{X+Y\} = E\{X\} + E\{Y\}$ and $E\{XY\} = E\{X\} \cdot E\{Y\}$, i.e. $E\{X+Y\} = 3.5 + 3.5 = 7$ and $E\{XY\} = 3.5 \cdot 3.5 = 12.25$.