

This week, practice working with special matrices:

- Question A:** (*Identity Matrix*) Determine n and m so that $I_n A = A$ and $A I_m = A$, where
- (i) A is (2×3) .
 - (ii) A is (5×7) .

Question B: (*Upper Triangular Matrix*) A (3×3) matrix $T = (t_{ij})$ is called upper triangular if $T_{ij} = 0$ whenever $i > j$. In other words, T has the form

$$T = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & 0 & t_{33} \end{bmatrix}.$$

If A and B are upper triangular (3×3) matrices, verify that the product AB is also upper triangular.

[*This also works for lower-triangular matrices!*]

Question C: (*Derivative Matrix*) Using the Derivative Matrix, calculate the fourth derivative of $x^7 + 3x^6 + 5x^4 - 2x^3 + 7x + 1$.

Question D: (*Rotation Matrix*) What vector do you get when you rotate the vector $\mathbf{v} = [1, \sqrt{3}]$ by $\theta = 5\pi/6$?