My name is:

Math 308U

Quiz 2

October 27, 2004

Question 1: (10 points) Determine whether the following set of vectors is linearly independent or linearly dependent.

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix},$$

Solve the system $V\mathbf{x} = \mathbf{0}$. If there is only the trivial solution then the vectors are linearly independent. Row reducing the matrix

$$\begin{bmatrix} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{bmatrix}$$

yields the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

so there is only the trivial solution.

Question 2: (10 points) Using the inverse of the appropriate 2×2 matrix, find the solution to the following system of equations.

$$\begin{array}{rcrcr} 3x_1 & - & 7x_2 & = & -4 \\ -6x_1 & + & 13x_2 & = & 1 \end{array}$$

We want to solve the system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 3 & -7 \\ -6 & 13 \end{bmatrix}$. We calculate $\Delta = 3 \cdot 13 - (-6) \cdot (-7) = -3$, so A^{-1} equals

$$A^{-1} = \frac{1}{-3} \begin{bmatrix} 13 & 7 \\ 6 & 3 \end{bmatrix}.$$

Calculating $A^{-1}\mathbf{b}$ gives our answer,

$$\mathbf{x} = \begin{bmatrix} 15 \\ 7 \end{bmatrix}.$$

Question 3: Quick Answer Questions (QuAQ's)

(a) (3 points) Give an example of a (3×3) symmetric matrix with non-zero entries. [*Do not use variables.*]

$$\begin{bmatrix} 1 & 6 & 5 \\ 6 & 2 & 4 \\ 5 & 4 & 3 \end{bmatrix}$$

(b) (3 points) Give a reason why the vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$ are linearly dependent.

There are more vectors than variables ($m = 2 < 3 = n$) so the vectors are linearly dependent.

(c) (3 points) Give a reason why the vectors $\mathbf{w}_1 = \begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{w}_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ are linearly dependent.

A set of vectors is linearly dependent if the zero vector is one of the vectors.

(d) (3 points) If the matrix A has $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$ as its columns, is A invertible?

A is invertible if and only if its columns are linearly independent. Therefore A is not invertible.

(e) (3 points) Simplify the expression $(BA)^T(ACB^T)^{-1}(D^{-1}A)^T$, where A, B, C, and D are $n \times n$ matrices.

$$A^T C^{-1} A^{-1} A^T (D^T)^{-1}$$