Math 308U

Quiz 1 Solutions

October 20, 2004

Question 1: (15 points)

(a) (10 points) Use Gaussian Elimination to find the General Solution to the following system of equations.

$$\begin{array}{rrrrrrrrrr} & - & 4x_3 & + & 8x_4 & + & 18x_5 & = & 3 \\ 2x_1 & + & 4x_3 & - & 6x_4 & - & 12x_5 & = & 4 \\ 3x_1 & + & 5x_3 & - & 10x_4 & - & 14x_5 & = & 8 \end{array}$$

Unfortunately I made a mistake and there were excessive fractions in the solving of this. The augmented matrix that comes from this system of equations is (2 points)

$$\left[\begin{array}{rrrrrr} 0 & 0 & -4 & 8 & 18 & 3 \\ 2 & 0 & 4 & -6 & -12 & 4 \\ 3 & 0 & 5 & -10 & -14 & 8 \end{array}\right]$$

Getting into reduced echelon form goes something like this (5 points):

$$\overset{R_1 \leftrightarrow R_2}{\rightarrow} \overset{\frac{1}{2}R_1}{\rightarrow} \left[\begin{array}{rrrrrr} 1 & 0 & 2 & -3 & -6 & 2 \\ 0 & 0 & -4 & 8 & 18 & 3 \\ 3 & 0 & 5 & -10 & -14 & 8 \end{array}\right] \overset{R_3+(-3)R_1}{\rightarrow} \overset{R_2 \leftrightarrow R_1}{\rightarrow} \overset{(-1)R_2}{\rightarrow} \left[\begin{array}{rrrrrr} 1 & 0 & 2 & -3 & -6 & 2 \\ 0 & 0 & 1 & 1 & -4 & -2 \\ 0 & 0 & -4 & 8 & 18 & 3 \end{array}\right]$$

$$\overset{R_3+4R_1}{\rightarrow} \overset{R_1+(-2)R_2}{\rightarrow} \left[\begin{array}{rrrrrr} 1 & 0 & 0 & -5 & -2 & 6 \\ 0 & 0 & 1 & 1 & -4 & -2 \\ 0 & 0 & 0 & 12 & 2 & -5 \end{array}\right] \overset{\frac{1}{12}R_3}{\rightarrow} \overset{R_1+5R_3}{\rightarrow} \overset{R_2+(-1)R_3}{\rightarrow} \left[\begin{array}{rrrrrr} 1 & 0 & 0 & 0 & \frac{17}{6} & \frac{47}{12} \\ 0 & 0 & 1 & 0 & \frac{-25}{6} & \frac{-19}{12} \\ 0 & 0 & 0 & 1 & \frac{1}{6} & \frac{-5}{12} \end{array}\right].$$

Reverting into equation form gives (2 points)

$$\begin{array}{rcrrr} x_1 & = & \frac{47}{12} & - & \frac{17}{6}x_5 \\ x_3 & = & -\frac{19}{12} & + & \frac{25}{6}x_5 \\ x_4 & = & -\frac{5}{12} & - & \frac{1}{6}x_5 \end{array}$$

with x_2 and x_5 free. (1 point)

(b) (5 points) Give a Particular Solution to the same system of equations **where each independent variable is non-zero**.

Choose your favorite NON-ZERO values for x_2 and x_5. Then plug into the equations and you get a particular solution.

Question 2: (8 points) For the following electrical system, find the system of equations that gives the current in each wire when solved. [**You need not solve the system.**]

There are only three wires. The left wire I_1, the central wire I_2, and the right wire I_3. Giving the clockwise orientation to both I_1 and I_3 and the downward orientation to I_2 yields the following system of equations

$$\begin{array}{rcrcrcl} I_1 & - & I_2 & - & I_3 & = & 0 \\ & & 6I_2 & & & = & 2 \\ 11I_1 & & & & & = & 8 \end{array}$$

Question 3: Quick Answer Questions (QuAQ's)

(a) (3 points) Give an example of the augmented matrix for a (2×3) homogeneous system of equations.

$$\left[\begin{array}{cccc} * & * & * & 0 \\ * & * & * & 0 \end{array}\right]$$

(b) (3 points) Give an example of a (3×2) non-homogeneous system of equations.

$$\left[\begin{array}{cccc} * & * & * & 0 \\ * & * & * & 0 \end{array}\right]$$

(c) (3 points) Which of the above systems (3a or 3b) is guaranteed to have infinitely many solutions?

$$\begin{array}{rcrcl} ax_1 & + & bx_2 & = & 42 \\ cx_1 & + & dx_2 & = & \sqrt{7} \\ ex_1 & + & ex_2 & = & 6.5 \end{array}$$

(d) (3 points) What are the possibilities for the number of solutions for the *other* system?

ALL ARE POSSIBLE!