

The **other** type of hyperbola

When you are trying to massage an equation into the standard equation of a hyperbola of the form $(x - c)(y - d) = E$, one way to do it is as follows:

1. Before you get started, the warning sign that you will need to do this is that there is a cross term xy , perhaps with a coefficient.
2. Divide the entire equation by the coefficient of xy (if it exists), and move the constant term to the other side of the equation.
3. Determine c and d . We want our equation to look like $(x - c)(y - d) = E$. If we were to expand this product, we would have an equation of the form $xy - dx - cy + cd = E$. So the coefficients of x and y are actually d and c , respectively. (Not the other way around!)
4. Add cd to both sides of the equation, and then factor the $xy - dx - cy + cd$ as $(x - c)(y - d)$.
5. The center of the graph is at (c, d) , with horizontal and vertical asymptotes emanating from the center. If E is positive, the curves are in the upper-right and lower-left quadrants. Otherwise, the curves are in the upper-left and lower-right quadrant.

Here is an example:

1. Determine the standard form of the following equation: (Note the xy cross term.)

$$5xy + 10x - 25y + 30 = 0$$

2. Divide by 5.

$$xy + 2x - 5y + 6 = 0$$

Then move the 6 to the other side of the equation.

$$xy + 2x - 5y = -6$$

3. Since the coefficient of x is 2 and the coefficient of y is -5 , then $d = 2$ and $c = -5$.
4. So, add $cd = -10$ to both sides of the equation, and factor.

$$xy + 2x - 5y - 10 = -6 - 10$$

$$(x - 5)(y + 2) = -16$$

5. The center is at $(5, -2)$, and the curves are on the upper-left and lower-right sections defined by the asymptotes.