Solutions:

1. $\lim_{x\to -2} (x^2-2x+1) = (-2)^2-2(-2)+1 = 9$

$\lim_{x\to -1} \dfrac{x^2-4x-5}{x+1} = \lim_{x\to -1} \dfrac{(x-5)(x+1)}{(x+1)} = \lim_{x\to -1} ($

$\lim_{x\to 3^+} \sqrt[4]{3-x}$ D.N.E. because $x\to 3^+$ means $x>3$, and $\sqrt[4]{3-x}$ is undefined for $x>3$.

$\lim_{t\to 4} \dfrac{t-4}{\sqrt{t}-2} = \lim_{t\to 4} \dfrac{(\sqrt{t}-2)(\sqrt{t}+2)}{(\sqrt{t}-2)} = \lim_{t\to 4} (\sqrt{t}+2) = (\sqrt{4}+2) = 4$. An alternative solution is:

$\lim_{t\to 4} \dfrac{t-4}{\sqrt{t}-2} = \lim_{t\to 4} \dfrac{(t-4)(\sqrt{t}+2)}{(\sqrt{t}-2)(\sqrt{t}+2)} = \lim_{t\to 4} \dfrac{(t-4)(\sqrt{t}+2)}{(t-4)} = \lim_{t\to 4} (\sqrt{t}+2) = 4$

$\lim_{x\to 0} \dfrac{|\cos x|}{\cos x} = \lim_{x\to 0} \dfrac{\cos x}{\cos x}$ (because $\cos x > 0$ for all x "close" to 0) $= \lim_{x\to 0} 1 = 1$

$\lim_{x\to -2} \dfrac{6}{x+2}$ DNE because $\lim_{x\to -2^-} \dfrac{6}{x+2} = -\infty$ and $\lim_{x\to -2^+} \dfrac{6}{x+2} = +\infty$. LHL $\neq$ RHL

$\lim_{x\to 4} \dfrac{x^2-16}{\sqrt{x+5}-3} = \lim_{x\to 4} \dfrac{(x+4)(x-4)(\sqrt{x+5}+3)}{(\sqrt{x+5}-3)(\sqrt{x+5}+3)} = \lim_{x\to 4} \dfrac{(x+4)(x-4)(\sqrt{x+5}+3)}{(x+5-9)} = \lim_{x\to 4} (x+4)(\sqrt{x+5}$
$=8$

$\lim_{x\to 0} \dfrac{1-\frac{2}{x}}{1+\frac{3}{x}} = \lim_{x\to 0} \dfrac{x-2}{x+3} = -\dfrac{2}{3}$

$\lim_{x\to \pi/2} \dfrac{1-\cos x}{x^2} = \dfrac{1-\cos(\pi/2)}{(\pi/2)^2} = \dfrac{1-0}{\pi^2/4} = \dfrac{4}{\pi^2}$

$\lim_{x\to 0} \dfrac{\tan^2 x}{2x\sin x} = \lim_{x\to 0} \dfrac{\frac{\sin^2 x}{\cos^2 x}}{2x\sin x} = \lim_{x\to 0} \dfrac{\sin^2 x}{\cos^2 x\cdot 2x\cdot \sin x} = \lim_{x\to 0} \dfrac{1}{\cos^2 x}\cdot\dfrac{1}{2}\cdot\dfrac{\sin x}{x}\cdot\dfrac{\sin x}{\sin x} = \dfrac{1}{1}\cdot\dfrac{1}{2}\cdot 1$

$\lim_{x\to 0} \cot(7x)\sin(2x) = \lim_{x\to 0} \dfrac{\cos(7x)}{\sin(7x)}\cdot\sin(2x) = \lim_{x\to 0} \cos(7x)\cdot\dfrac{(7x)}{\sin(7x)}\cdot\dfrac{\sin(2x)}{2(x)}\cdot\dfrac{(2x)}{(7x)}$

$= \lim_{x\to 0} \cos(7x)\cdot 1\cdot 1\cdot \dfrac{2}{7} = \dfrac{2}{7}$

$\lim_{x\to 0} \dfrac{x+\sin(2x)}{x} = \lim_{x\to 0}\left[\dfrac{x}{x} + \dfrac{\sin(2x)}{x}\right] = \lim_{x\to 0}\left[1 + \dfrac{2\sin(2x)}{(2x)}\right] = 1+2 = 3$

$\lim_{x\to \infty} \dfrac{3x^2-10x+1}{x^2+20x+15} = \lim_{x\to\infty} \dfrac{3-\frac{10}{x}+\frac{1}{x^2}}{1+\frac{20}{x}+\frac{15}{x^2}} = \dfrac{3}{1} = 3$

$\lim_{x\to -\infty} \dfrac{2x}{\sqrt{9x^2+1}} = \lim_{x\to -\infty} \dfrac{\frac{2x}{-x}}{\frac{\sqrt{9x^2+1}}{-x}} = \lim_{x\to -\infty} \dfrac{-2}{\sqrt{9+\frac{1}{x^2}}} = \dfrac{-2}{\sqrt{9}} = -\dfrac{2}{3}$

$\lim_{x\to\infty} (x-\sqrt{x^2+x}) = \lim_{x\to\infty} \dfrac{(x-\sqrt{x^2+x})(x+\sqrt{x^2+x})}{(x+\sqrt{x^2+x})} = \lim_{x\to\infty} \dfrac{x^2-(x^2+x)}{(x+\sqrt{x^2+x})} = \lim_{x\to\infty} \dfrac{-x}{x+\sqrt{x^2+x}}$

$= \lim_{x\to\infty} \dfrac{-1}{1+\sqrt{1+\frac{1}{x}}} = \dfrac{-1}{1+1} = -\dfrac{1}{2}$

$f(x) = \begin{cases} 4x & \text{if } x \le -1 \\ x+1 & -1<x<1 \\ 4 & x=1 \\ 3x^2-1 & x>1 \end{cases}$

$\left.\begin{aligned} \lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-} 4x = -4 \\ \lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+} (x+1) = 0 \end{aligned}\right\}$ so $\lim_{x\to -1} f(x)$ DNE because LHL $\neq$ RHL

$\lim_{x\to 0} f(x) = \lim_{x\to 0} (x+1) = 1$

$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} (x+1) = 2 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} (3x^2-1) = 2 \end{aligned}\right\}$ so $\lim_{x\to 1} f(x) = 2$

#2. $f(x) = |x-5|$

$= \begin{cases} -(x-5) & \text{if } x<5 \\ (x-5) & \text{if } x \ge 5 \end{cases}$

If $x \neq 5$, f is continuous because the function is described by a polynomial.

If $x=5$, consider $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} (x-5) = 0$ and $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} -(x-5) = 0$, and $f(5)=0$.

So, f is continuous at 5.

3. $g(x)=\begin{cases} x^2+1 & \text{if } x\le 2 \\ \dfrac{x^2-4}{x+2}+B & \text{if } x>2 \end{cases}$

$g(x)$ is continuous at 2 if $\lim_{x\to 2} g(x) = g(2)$.

$g(2)=5$

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-}(x^2+1) = 5$

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+}\left[\dfrac{x^2+4}{x+2}+B\right] = \dfrac{4-4}{2+2}+B = 0+B = B.$

So, we need $B=5$ for $\lim_{x\to 2} g(x)$ to be 5.

4. $f(x)=\begin{cases} |1-x| & \text{if } x\le 2 \\ \sqrt{2x} & \text{if } x>2 \end{cases}$

f is not differentiable at 1 because equivalently, $f(x)=\begin{cases} +(1-x) & \text{if } x<1 \\ -(1-x) & \text{if } 1\le x\le 2 \\ \sqrt{2x} & \text{if } x>2 \end{cases}$. $f'(x)=\begin{cases} -1 & \text{if } x<1 \\ 1 & 1<x<2 \\ \dfrac{1}{\sqrt{2x}} & x>2 \end{cases}$ Since LHD $=-1$ and RHD $=1$ $f'(1)$ D.N.E.

f is discontinuous at 2 because LHL $\neq$ RHL

$LHL = \lim_{x\to 2^-}|1-x| = |2-1| = 1$ $\quad RHL = \lim_{x\to 2^+}\sqrt{2x} = \sqrt{4} =$

Because f is not continuous at 2, it is not differentia at 2

5. $f(x)=\dfrac{x}{x-1}$. $f'(2)=\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\left[\dfrac{\dfrac{(2+h)}{(2+h)-1}-2}{h}\right] = \lim_{h\to 0}\left[\dfrac{\dfrac{2+h}{1+h}-\dfrac{2(1+}{(1+}}{h}\right.$

$= \lim_{h\to 0}\dfrac{2+h-2-2h}{h(1+h)} = \lim_{h\to 0}\dfrac{-h}{h(1+h)} = \lim_{h\to 0}\dfrac{-1}{1+h} = -1.$

6. $f(x)=\sqrt{x-1}$ $\quad f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\dfrac{\sqrt{(x+h)-1}-\sqrt{x-1}}{h} = \lim_{h\to 0}\dfrac{(\sqrt{(x+h)-1}-\sqrt{x-1})(\sqrt{x+h-1}}{h(\sqrt{x+h-1}+\sqrt{}}$

$= \lim_{h\to 0}\dfrac{(x+h)-1-(x-1)}{h(\sqrt{x+h-1}+\sqrt{x-1})} = \lim_{h\to 0}\dfrac{h}{h(\sqrt{x+h-1}+\sqrt{x-1})} = \lim_{h\to 0}\dfrac{1}{\sqrt{x+h-1}+\sqrt{x-1}} = \dfrac{1}{\sqrt{x-1}+\sqrt{x-1}} = \dfrac{1}{2\sqrt{x-}}$

7. $\lim_{x\to \pi/4}\dfrac{\tan x - 1}{x-\pi/4}$ $\quad$ let $a=\pi/4$. Then $\tan a = \tan\pi/4 = 1$. This limit then has the form

$\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a} = f'(a)$. where $f(x)=\tan x$.

Since $\dfrac{d}{dx}\tan x = \sec^2 x$, we get $\lim_{x\to\pi/4}\dfrac{\tan x-1}{x-\pi/4} = \sec^2(\pi/4) = (\sqrt{2})^2 = 2$

8. (a) $f'(x) = 3(2x^2-3)^2\cdot 4x\,(5-2x)^4 + 4(5-2x)^3(-2)(2x^2-3)^3$

(b) $f'(x) = \cos(\sin x^2)\cos(x^2)\cdot 2x$

(c) $f'(x) = (\cos x)(\cos x) + (-\sin x)(\sin x)$

(d) $f(x) = x^2(x^2+1)^{-1/2}$ $\quad f'(x) = 2x\,(x^2+1)^{-1/2} + -\tfrac{1}{2}(x^2+1)^{-3/2}\cdot 2x\cdot x^2$

or $f(x) = \dfrac{x^2}{(x^2+1)^{1/2}}$ $\quad f'(x) = \dfrac{2x(x^2+1)^{1/2} - \frac{1}{2}(x^2+1)\,2x\cdot x^2}{[(x^2+1)^{1/2}]^2}$

(e) $f'(x) = \dfrac{2\sin x\cos x\cdot\cos x - (-\sin x)\sin^2 x}{(\cos x)^2}$

(f) $f(x) = (\tan x)(3x^{-1}+\tfrac{1}{3}x)$ $\quad f'(x) = \sec^2 x\,(3x^{-1}+\tfrac{1}{3}x) + (-3x^{-2}+\tfrac{1}{3})(\tan x)$

(g) $f'(x) = \dfrac{(3ax^2+2bx)(x^a-x) - (ax^{a-1}-1)(ax^3+bx^2-5)}{(x^a-x)^2}$

9. $f(x) = x^{13} + \frac{1}{x} + \sin(3x)$

$f'(x) = 13x^{12} - x^{-2} + \cos(3x)\cdot 3$

$f''(x) = 13\cdot 12x^{11} + 2x^{-3} - \sin(3x)\cdot 3\cdot 3$

$f'''(x) = 13\cdot 12\cdot 11x^{10} + (-3)(2)x^{-4} - \cos(3x)\cdot 3\cdot 3\cdot 3$

$f^{(4)}(x) = \underbrace{13\cdot 12\cdot 11\cdot 10x^{9}} + (-4)(-3)(2)x^{-5} + \underbrace{\sin(3x)\,3\cdot 3\cdot 3\cdot 3}$

$\vdots$

$f^{(27)}(x) = 0 + (-1)27!\,x^{-28} + -\cos(3x)\cdot 3^{27}$

10. $f(x) = \frac{1}{(3x-4)} = (3x-4)^{-1}$

$f'(x) = -(3x-4)^{-2}\cdot 3$

$f''(x) = 2(3x-4)^{-3}\cdot 3\cdot 3$

$f'''(x) = (-3)(2)(3x-4)^{-4}\cdot 3\cdot 3\cdot 3$

$f^{(4)}(x) = (-4)(-3)(2)(3x-4)^{-5}\cdot 3\cdot 3\cdot 3\cdot$

$\vdots$

$f^{(47)}(x) = -47!\,(3x-4)^{-48}\cdot 3^{47}$

11. $u(x) = f\sqrt{x}$, so $u'(x) = f'(\sqrt{x})\cdot \frac{1}{2}x^{-1/2}$ (chain rule). So, $u'(4) = f'(\sqrt{4})\cdot\frac{1}{2\sqrt{4}} = f'(2)\cdot$

$= 1\cdot\frac{1}{4} =$

$v(x) = \sqrt{f(x)}$, so $v'(x) = \frac{1}{2}(f(x))^{-1/2}f'(x)$ (chain rule). So $v'(4) = \frac{1}{2}(f(4))^{-1/2}f'(4) = \frac{1}{2}\cdot\frac{1}{3}\cdot 5 = 5/6$

12. $yx^2 - y^2x + 2y^2 + x^2 = 1$. When $x=1, y=-1$, we get $(-1)(1)^2 - (-1)^2(1) + 2(-1)^2 + (1)^2 = -1-1+$
so pt. $(+1,-1)$ is on the graph.

$\frac{dy}{dx}x^2 + 2xy - 2y\frac{dy}{dx}x + -y^2 + 4y\frac{dy}{dx} + 2x = 0$

$\frac{dy}{dx}(x^2 - 2y + 4y) = -2xy + y^2 - 2x$

$\frac{dy}{dx} = \frac{-2xy + y^2 - 2x}{x^2 + 2y}$

$\frac{dy}{dx}$ at $(1,-1)$ becomes $\frac{-2(1)(-1) + (-1)^2 - 2(1)}{(1)^2 + 2(-1)} =$

So slope of tangent line is -1; point is $(1,-$

This gives equation: $y - -1 = -1(x-1) \Rightarrow y =$

13. $(x+y)^2 + 3x + y = -6$

$2(x+y)(1 + \frac{dy}{dx}) + 3 + \frac{dy}{dx} = 0$

$2(x+y) + 2(x+y)\frac{dy}{dx} + 3 + \frac{dy}{dx} = 0$

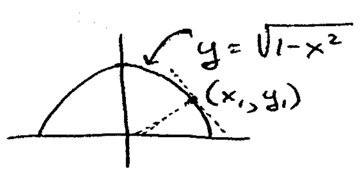
$\frac{dy}{dx}[2x + 2y + 1] = -3 - 2x - 2y$

$\frac{dy}{dx} = \frac{-3-2x-2y}{2x+2y+1}$. At pt $(-3,2)$, $\frac{dy}{dx} = 1$

So, equation of tangent line is $y - 2 = 1(x+3) \Rightarrow y = x + 5$

14. $f(x) = x^2 + 1$ $f'(x) = 2x$. Suppose the point $(a, f(a))$ is on the line tangent to f that goes through the point $(0,-2)$. We know the slope of this line is $f'(a) = 2a$. The slope is also $\frac{\Delta y}{\Delta x} = \frac{f(a)-2}{a-0} = \frac{a^2+1+2}{a}$. So, since $2a = \frac{a^2+3}{a}$, we get: $2a^2 = a^2 + 3 \Rightarrow$ $a^2 = 3 \Rightarrow a = \pm\sqrt{3}$. So, the points must be $(\sqrt{3},4)$ and $(-\sqrt{3},4)$.

15.



Suppose l_1 is the line tangent to the curve at (x_1, y_1). The slope of l_1 is y' evaluated at $x = x_1$.

$y' = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{1-x^2}} = -\frac{x}{y}$. At (x_1, y_1) we get $\frac{-x_1}{y_1}$

The slope of line from the origin to the point (x_1, y_1) is $\frac{\Delta y}{\Delta x} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$. Since the slopes of these 2 lines are negative reciprocals, the lines must be perpendicular.

16. Estimate $\sqrt{99.7}$. Use $f(x)=\sqrt{x}=x^{1/2}$; $a=100$. $f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$f(x) \approx f(a)+f'(a)(x-a)$.

$\sqrt{99.7} \approx \sqrt{100}+\frac{1}{2\sqrt{100}}(99.7-100) = 10+\frac{1}{20}(-.3) = 10-\frac{3}{200} = 10-.015 = 9.98$

Estimate $\sqrt{100.3}$

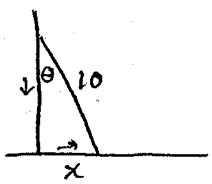
$\sqrt{100.3} \approx \sqrt{100}+\frac{1}{2\sqrt{100}}(100.3-100) = 10+\frac{1}{20}(+.3) = 10+\frac{3}{200} = 10+.015 = 10.0$

Estimates will always be $\pm\frac{1}{20}$ times the distance between our estimated number and 100. This is because $\frac{1}{20}$ is the slope ($f'(100)$) of the line tangent to $f(x)$ at $x=$

17. Volume: $V=s^3$ where s = side of cube. $V(4)=4^3=64$. $V'=3s^2$

$V(3.8) \approx (4)^3+3(4)^2(3.8-4) = 64+48(-.2) = 64-.96 = 63.04$

18.



Know: $\frac{dx}{dt}=2$ ft/sec.

Want $\frac{d\theta}{dt}$ when $\theta=\pi/4$ rad.

$\sin\theta=\frac{x}{10}$

$\frac{d}{dt}\sin\theta=\frac{d}{dt}\left(\frac{x}{10}\right)$

$\cos\theta\,\frac{d\theta}{dt}=\frac{1}{10}\frac{dx}{dt}$

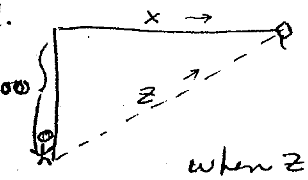
at $\theta=\pi/4$

$\cos(\pi/4)\frac{d\theta}{dt}=\frac{1}{10}(2)$

$\frac{\sqrt{2}}{2}\frac{d\theta}{dt}=\frac{1}{5}$

$\frac{d\theta}{dt}=\frac{2}{5\sqrt{2}}$ rad./se

19.



Know: $\frac{dz}{dt}=3$ ft/sec.

Want: $\frac{dx}{dt}$ when $z=125$ ft

when $z=125$, $x=\sqrt{125^2-100^2}=75$

$x^2+100^2=z^2$

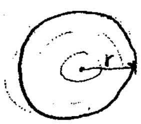
$\frac{d}{dt}(x^2+100^2)=\frac{d}{dt}z^2$

$2x\frac{dx}{dt}+0=2z\frac{dz}{dt}$

$\cancel{2}(75)\frac{dx}{dt}=\cancel{2}(125)3$

$\frac{dx}{dt}=\frac{125\cdot 3}{75}=5$ f

20.



Know: $\frac{dr}{dt}=1.5$ ft/sec.

(a) want $\frac{dA}{dt}$ when $r=30$ ft.

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

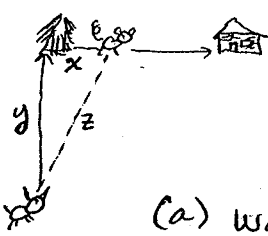
$\frac{dA}{dt}=2\pi(30)(1.5)$
$=60\pi$ ft²/sec.

(b) Want $\frac{dA}{dt}$ when $t=120$ sec.

When $t=120$ sec., $r=(1.5)(120)=180$ ft.

$\frac{dA}{dt}=2\pi(180)(1.5)$
$=540\pi$ ft²/sec.

21.



Know: $\frac{dy}{dt}=-96$ meters/min (negative because y is decreasing)

$\frac{dx}{dt}=80$ meters/min (positive because x is increasing)

Let t = time pig begins to escape

(a) Want: Value of y when t=2

$y=320+(-96)(2)=128$ meters

(b) Want $\frac{dz}{dt}$ when $t=2$.

$x^2+y^2=z^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

When $t=2$, $x=2\cdot 80=160$

When $t=2$, $z=\sqrt{160^2+128^2}\approx 205$

$\frac{x\frac{dx}{dt}+y\frac{dy}{dt}}{z}=\frac{dz}{dt}$

$\frac{(160)(80)+(128)(-96)}{\sqrt{160^2+128^2}}=\frac{dz}{dt}\approx 2.5$ m/sec

22. $f(x) = -\frac{1}{2}x^4 + x^3 + \frac{1}{2}$ on $[-1, 2]$.

$f'(x) = -2x^3 + 3x^2$

$f'(x) = 0 = -2x^3 + 3x^2$

$0 = x^2(-2x+3)$

$x = 0, \frac{3}{2}$

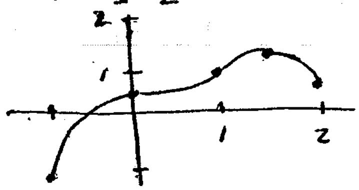
x	-1		0		$3/2$		2
f'		$+$	0	$+$	0	$-$	
f	-1	$\nearrow$	$\frac{1}{2}$	$\nearrow$	$\frac{43}{32}$	$\searrow$	$\frac{1}{2}$

$f''(x) = -6x^2 + 6x$
$= 6x(-x+1)$

$f''(x) = 0 = 6x(-x+1)$

$x = 0, x = 1$

x	-1		0		1		2
f''		$-$	0	$+$	0	$-$	
f		$\cap$	$\frac{1}{2}$	$\cup$	1	$\cap$	



f increases on $[-1, 3/2]$

f decreases on $(3/2, 2]$

f is concave up on $(0, 1)$

f is concave down on $(-1, 0) \cup (1, 2]$

f has loc max at $x = 3/2$

f has loc. min at $x = -1$ and $x = 2$

f has maximum value of $\frac{43}{32}$ at $x = 3$

f has minimum value of -1 at $x =$

f has points of inflection $(0, \frac{1}{2})$ and

23. $f(x) = \frac{1}{4}x^4 - x^3 + 5x - 7$

$f'(x) = x^3 - 3x^2 + 5$

$f''(x) = 3x^2 - 6x$

$f''(x) = 0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, x = 2$

x		0		2	
f''	$+$	0	$-$	0	$+$
f	$\cup$	-7	$\cap$	-1	$\cup$

f is concave up on $(-\infty, 0) \cup (2, \infty)$
concave down on $(0, 2)$

f has points of inflection $(0, -7)$ and $(2, -1)$

24. $f(x) = -2x^3 - 3x^2 + 12x - 5$ on $[0, 2]$

$f'(x) = -6x^2 - 6x + 12$

$f'(x) = 0 = -6(x^2 + x - 2)$

$0 = -6(x+2)(x-1)$

$x = 1, x = -2$ Note: $x = -2$ is NOT domain

x	0		1		2
f'		$+$	0	$-$	
f	-5	$\nearrow$	2	$\searrow$	-9

f increases on $[0, 1]$ and decreases on $[1,$

f has local maximum at $x = 1$

f has local minimums at $x = 0$ and

f has global maximum value of 2 at $x =$

f has global minimum value of -9 at

25. $f(x) = ax^2 + bx + c$ (Note: a, b, c are constants)

$f'(x) = 2ax + b$

$f''(x) = 2a$

When $2a > 0$, f is concave up, so when $a > 0$
f is concave up

When $2a < 0$, f is concave down, so when
$a < 0$, f is concave down

26. $f(x) = 2x^2 - \frac{x^3}{3} - 1 = 2x^2 - \frac{1}{3}x^3 - 1$

$f'(x) = 4x - x^2$

$f'(x) = 0 = x(4 - x)$ so $x = 0$ or $x = 4$

$f''(x) = 4 - 2x$

$f''(x) = 0 = 2(2 - x)$, so $x = 2$

x		0		4	
f'	$-$	0	$+$	0	$-$
f	$\searrow$	-1	$\nearrow$	$9\frac{2}{3}$	$\searrow$

x		2	
f''	$+$	0	$-$
	$\cup$	$4\frac{1}{3}$	$\cap$

f increases on $(0, 4)$

f decreases on $(-\infty, 0) \cup (4, \infty)$

f has loc. min at $x = 1$

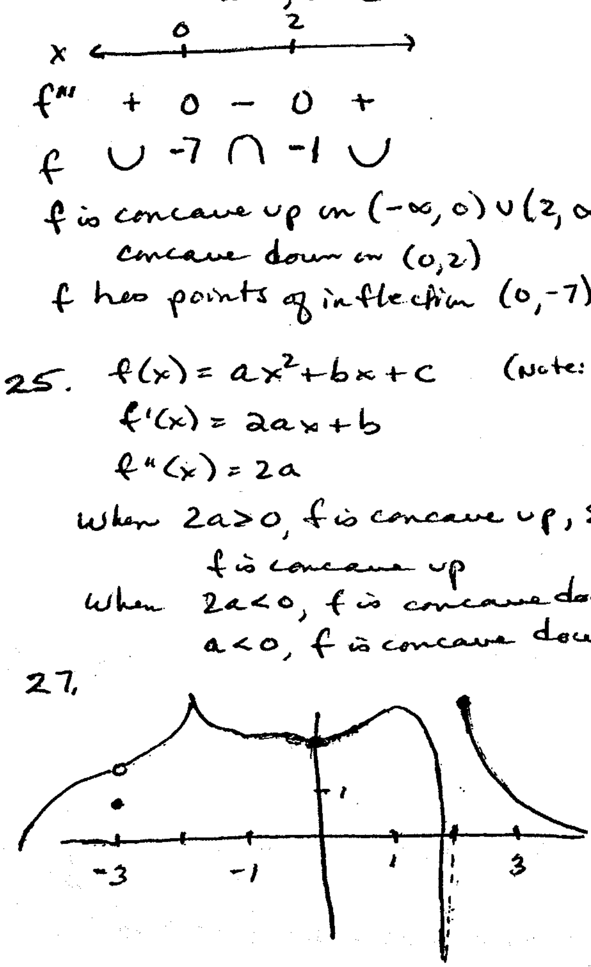
f has loc. max at $x = 4$

f is concave up on $(-\infty, 2)$

f is concave down on $(2, \infty)$

f has pt. of inflection $(2, 4\frac{1}{3})$

27.



32. $\frac{d}{dx}\int_5^x (t^2+3\cos t)\,dt = x^2+3\cos x$ $\qquad \frac{d}{dx}\int_7^{2x^3} (t^2+3\cos t)\,dt = [(2x^3)^2+3\cos(2x^3)]6$

$\frac{d}{dx}\int_5^8 (t^2+3\cos t)\,dt = 0$ because a definite integral is a constant.

$\frac{d}{dx}\int_x^9 (t^2+3\cos t)\,dt = -[x^2+3\cos x]$ $\qquad \frac{d}{dx}\int_{5x}^{\sin x} (t^2+3\cos t)\,dt =$

$= [(\sin x)^2+3\cos(\sin x)]\cos x - [(5x)^2+3\cos(5x)]\cdot$

33. $\int_0^1 \sin(\pi x)\,dx \begin{cases} \text{Let } u=\pi x & \text{if } x=1, u=\pi \\ du=\pi\,dx & \text{if } x=0, u=0 \\ \frac{1}{\pi}du = dx & \end{cases} \Rightarrow \int_0^\pi \frac{1}{\pi}\sin u\,du = \frac{-1}{\pi}\cos u \Big|_0^\pi$

$= -\frac{1}{\pi}(\cos\pi - \cos 0) = -\frac{1}{\pi}(-1-1) = \frac{2}{\pi}$

$\int \frac{1-4x^2+6x^4}{2x^2}\,dx = \int\left(\frac{1}{2}x^{-2}-2+3x^2\right)dx = \left(-\frac{1}{2}x^{-1}-2x+x^3\right)+C$

$\int_1^2 (1-x)\sqrt{2x-x^2}\,dx \begin{cases} \text{Let } u=2x-x^2 & \text{if } x=1, u=1 \\ du=(2-2x)dx & \text{if } x=2, u=0 \\ du=2(1-x)dx & \\ \frac{1}{2}du=(1-x)dx & \end{cases} \Rightarrow \int_1^0 \frac{1}{2}\sqrt{u}\,du = \int_1^0 \frac{1}{2}u^{1/2}du$

$= \frac{1}{3}u^{3/2}\Big|_1^0 = 0-\frac{1}{3} = \frac{-1}{3}$

$\int \frac{\sec^2 x}{\sqrt[3]{1+\tan x}}\,dx \begin{cases} \text{Let } u=(1+\tan x) \\ du=\sec^2 x\,dx \end{cases} \Rightarrow \int \frac{1}{\sqrt[3]{u}}\,du = \int u^{-1/3}du = \frac{3}{2}u^{2/3}+C$

$= \frac{3}{2}(1+\tan x)^{2/3} +$

$\int_{-2}^{3} |x^2-1|\,dx = \left|\int_{-2}^{-1}(x^2-1)dx\right| + \left|\int_{-1}^{1}(x^2-1)dx\right| + \left|\int_{1}^{3}(x^2-1)dx\right|$

$= \left|\left(\frac{1}{3}x^3-x\right)\Big|_{-2}^{-1}\right| + \left|\left(\frac{1}{3}x^3-x\right)\Big|_{-1}^{1}\right| + \left|\left(\frac{1}{3}x^3-x\right)\Big|_{1}^{2}\right|$

$= \left|\left(-\frac{1}{3}+1\right)-\left(-\frac{8}{3}+2\right)\right| + \left|\left(\frac{1}{3}-1\right)-\left(-\frac{1}{3}+1\right)\right| + \left|\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)\right|$

$\left|\frac{4}{3}\right| + \left|-\frac{4}{3}\right| + \left|\frac{4}{3}\right| = \frac{12}{3} = 4$

$\int (3x^3-6x)(x^2+x)\,dx = \int(3x^5+3x^4-6x^3-6x^2)\,dx = \left(\frac{1}{2}x^6+\frac{3}{5}x^5-\frac{3}{2}x^4-2x^3\right)+C$

$\int x^2\cos x\,dx \begin{cases} \text{Let } u=x^2 & du=2x\,dx \\ dv=\cos x\,dx & v=\sin x \end{cases} \Rightarrow \underbrace{x^2\sin x - \int 2x(\sin x)\,dx}$

For $\int 2x\sin x\,dx$, let $\begin{cases} u=2x & du=2dx \\ dv=\sin x\,dx & v=-\cos x \end{cases} \Rightarrow x^2\sin x - \left[-2x\cos x - \int -2\cos x\,dx\right]$

$= x^2\sin x + 2x\cos x - \int 2\cos x\,dx$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

(Go ahead,... check it).

#33 (continued)

$$\int_0^1 \frac{1}{1+u^2}\,du \quad \left\{\begin{array}{ll} \text{Let } u=\tan x & \text{If } u=1,\ 1=\tan x,\ x=\pi/4 \\ du=\sec^2 x\,dx & \text{If } u=0,\ 0=\tan x,\ x=0 \end{array}\right\} \Rightarrow \int_0^{\pi/4} \frac{1}{1+\tan^2 x}\sec^2 x\,dx$$

$$= \int_0^{\pi/4} \frac{\sec^2 x}{\sec^2 x}\,dx = \int_0^{\pi/4} 1\,dx = x\Big|_0^{\pi/4} = \pi/4 - 0 = \pi/4$$

34. Find where two curves intersect:

$$2(x^2-1) = x^2-2x+1$$
$$2x^2-2 = x^2-2x+1$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x=-3,\ x=1$$

Set up integral for area between 2 curves:

$$\int_{-3}^{1} \left|2(x^2-1)-(x^2-2x+1)\right|dx$$

$$= \left|\int_{-3}^{1}(x^2+2x-3)\,dx\right| = \left|\left(\tfrac{1}{3}x^3+x^2-3x\right)\Big|_{-3}^{1}\right|$$

$$= \left|\left(\tfrac{1}{3}+1-3\right)-\left(-9+9+9\right)\right| = \left|-1\tfrac{2}{3}-9\right| = 10\tfrac{2}{3}$$

35. In order to do this problem two ways we will need to solve the equations for each variable. We do this first:

①
$$y^2+2y=x-1$$
$$y^2+2y+1=x$$
$$(y+1)^2=x$$
$$y+1=\pm\sqrt{x}$$
$$y=\pm\sqrt{x}-1$$

$$1-y^2=x$$
$$-y^2=x-1$$
$$y^2=1-x$$
$$y=\pm\sqrt{1-x}$$

Then, find where they intersect.

② Using $x=y^2+2y+1$ and $x=1-y^2$ we get

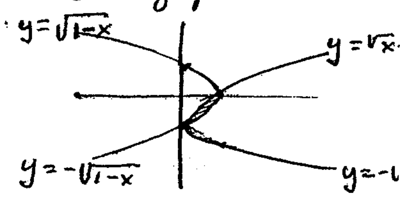
$$y^2+2y+1=1-y^2$$
$$2y^2+2y=0$$
$$2y(y+1)=0$$
$$y=0,\ y=-1$$

If $y=0$, $x=1$
If $y=-1$, $x=0$
So pts. of intersection are $(1,0)$ and $(0,-1)$

③ Area using y as independent variable:

$$\int_{-1}^{0}\left|(y^2+2y+1)-(1-y^2)\right|dy = \left|\int_{-1}^{0}(2y^2+2y)\,dy\right| = \left|\tfrac{2}{3}y^3+y^2\Big|_{-1}^{0}\right| = \left|(0)-\left(-\tfrac{2}{3}+1\right)\right| = \left|-\tfrac{1}{3}\right| = \tfrac{1}{3}$$

④ Area using x as independent variable requires selection of $\pm$, so look at graph:

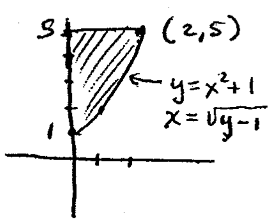


$$\int_0^1\left|(\sqrt{x}-1)-(-\sqrt{1-x})\right|dx = \left|\int_0^1(\sqrt{x}-1+\sqrt{1-x})\,dx\right|$$

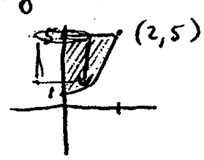
$$= \left|\int_0^1\left[x^{1/2}-1+(1-x)^{1/2}\right]dx\right| = \left|\left[\tfrac{2}{3}x^{3/2}-x-\tfrac{2}{3}(1-x)^{3/2}\right]\Big|_0^1\right|$$

$$= \left|\left(\tfrac{2}{3}-1-0\right)-\left(0-0-\tfrac{2}{3}\right)\right| = \left|-\tfrac{1}{3}+\tfrac{2}{3}\right| = \left|\tfrac{1}{3}\right| = \tfrac{1}{3}$$

36.



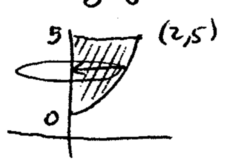
Using x as variable, we get a shell:



$$\int_0^2 2\pi x\left(5-(x^2+1)\right)dx = 2\pi\int_0^2 x(-x^2+4)\,dx = 2\pi\int_0^2(-x^3+4x)\,dx$$

$$= 2\pi\left[\left(-\tfrac{1}{4}x^4+2x^2\right)\Big|_0^2\right] = 2\pi\left[(-4+8)-(0)\right] = 2\pi\cdot 4 = 8\pi$$

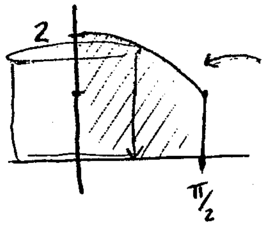
Using y as variable, we get disks:



$$\int_1^5 \pi(\sqrt{y-1})^2\,dy = \pi\int_1^5(y-1)\,dy = \pi\left(\tfrac{1}{2}y^2-y\right)\Big|_1^5 = \pi\left[\left(\tfrac{25}{2}-5\right)-\left(\tfrac{1}{2}-1\right)\right]$$

$$= \pi\left[\tfrac{24}{2}-4\right] = 8\pi$$

37.



$\leftarrow y = 1+\cos x$ We will use x as the variable, so we get a shell.

$$V = \int_0^{\pi/2} 2\pi x\,(1+\cos x)\,dx = 2\pi \int_0^{\pi/2} x\,(1+\cos x)\,dx$$

To find antiderivative for $\int x(1+\cos x)\,dx$ we use integration by parts:

$\int x\,(1+\cos x)\,dx$ Let $u = x$ $du = dx$

$dv = (1+\cos x)\,dx$ $v = (x+\sin x)$

$= x(x+\sin x) - \int (x+\sin x)\,dx$

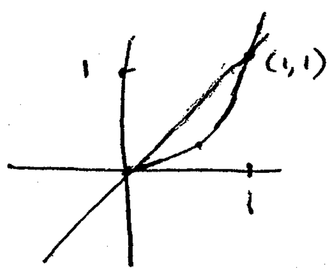
$= x^2 + x\sin x - \frac{1}{2}x^2 + \cos x$

$$2\pi\left[x^2 + x\sin x - \tfrac{1}{2}x^2 + \cos x\right]\Big|_0^{\pi/2}$$

$$= 2\pi\left[\left(\tfrac{\pi^2}{4} + \tfrac{\pi}{2}\cdot 1 - \tfrac{\pi^2}{8} + 0\right) - (0+0-0+1)\right]$$

$$= 2\pi\left[\tfrac{\pi^2}{8} + \tfrac{\pi}{2} - 1\right] = \tfrac{\pi^3}{4} + \pi^2 - 2\pi$$

38.

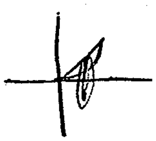


line $y = x$; $x = y$

parabola half $y = x^2$; $x = \sqrt{y}$

intersect at $(0,0)$, $(1,1)$.

(a)



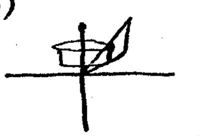
Around x-axis using x as indep. variable gives washer.

$$\int_0^1 \left[\pi(x)^2 - \pi(x^2)^2\right]dx = \pi\int_0^1 (x^2 - x^4)\,dx = \pi\left[\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right]\Big|_0^1 = \pi\left[\left(\tfrac{1}{3}-\tfrac{1}{5}\right) - 0\right] = \tfrac{2}{15}\pi$$

Around x-axis using y as indep. variable gives shell.

$$\int_0^1 2\pi y\,(\sqrt{y} - y)\,dy = 2\pi\int_0^1 (y^{3/2} - y^2)\,dy = 2\pi\left[\left(\tfrac{2}{5}y^{5/2} - \tfrac{1}{3}y^3\right)\Big|_0^1\right] = 2\pi\left[\left(\tfrac{2}{5} - \tfrac{1}{3}\right) - (0)\right] = \tfrac{2}{15}$$

(b)



Around y-axis using x as indep. variables gives shell.

$$\int_0^1 2\pi x(x - x^2)\,dx = 2\pi\int_0^1 (x^2 - x^3)\,dx = 2\pi\left(\tfrac{1}{3}x^3 - \tfrac{1}{4}x^4\right)\Big|_0^1 = 2\pi\left[\left(\tfrac{1}{3} - \tfrac{1}{4}\right) - 0\right] = \pi/6$$

Around y-axis using y as indep. variable gives washer:

$$\int_0^1 \left(\pi(\sqrt{y})^2 - \pi y^2\right)dy = \pi\int (y - y^2)\,dy = \pi\left[\left(\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right)\Big|_0^1\right] = \pi\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \pi/6$$

39. (a) False, Differentiable $\Rightarrow$ Continuous. NOT Continuous $\Rightarrow$ Diff. Example: $y =$ [illegible] at $x=0$

(b) True

40. E, D, B, A, G, H