Practice Quiz, Week 13
November 22, 2006

1. (7 pts) The below curve is defined on the interval $[1, 7]$.

2

1.5

1

0.5

0

1   2   3   4   5   6   7   8
        x
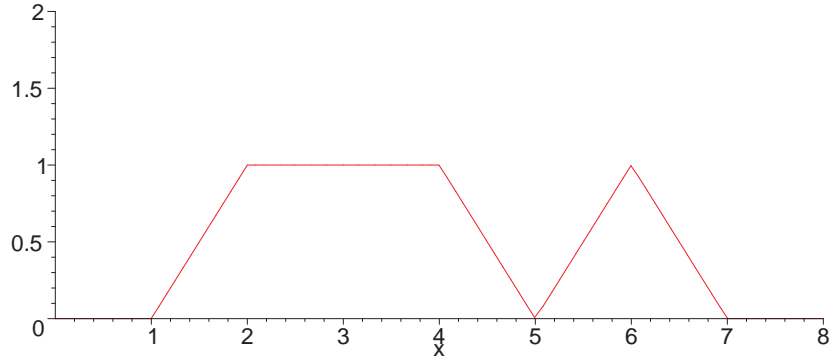
(a) What is the area under the curve on the interval $[1, 7]$?

(b) Approximate this area by a Riemann sum using left endpoints when $n = 3$.
Write out the sum you are taking before evaluating it!

(c) Approximate this area by a Riemann sum using midpoints when $n = 6$.
Write out the sum you are taking before evaluating it!

2. (3 pts) Fill in the blank. The right-hand Riemann sum is a **lower approximation** for the

   area under the curve $f(x)$ if f is _____. (*Hint: Draw a picture!*)

3. (4 pts) The function $f(x) = \sin^2 x + x$ is increasing on the interval $[\pi/2, \pi]$. Use Problem 23.3c to find upper and lower bounds on the integral

$$\int_{\pi/2}^{\pi} (\sin^2 x + x)\, dx$$

4. (6 pts) Using summation notation, give the definition of the definite integral $\displaystyle\int_{-1}^{1} x^3\, dx$.

5. (4 pts each) Take the following definite integrals.

   (a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin t \, dt =$

   (b) $\int_{0}^{3} |x^2 - 1| \, dx =$

6. (4 pts) Use the Fundamental Theorem of Calculus to solve

   $$\frac{d}{dx} \int_{1}^{\sqrt{x}} \frac{2x^2}{\tan x}.$$

7. (6 pts) It's pouring down rain! The rate at which water is entering my rain barrel is given by $r(t) = 2 - t^{1/2}$ inches/hour for $0 \le t \le 4$. Determine the how full the rain barrel is after four hours.