Girth and diameter

Definition: The **girth** of a graph G, denoted $\mathrm{g}(G)$, is the length of the shortest cycle contained in G. If no cycles exist, $\mathrm{g}(G) = \infty$.

Definition: Let $x, y \in V(G)$. The **distance** from x to y, denoted $d(x, y)$, is the length of the shortest path from x to y.

If no path exists between x and y, then $d(x, y) = \infty$.

Definition: The **diameter** of a graph G, denoted $\mathrm{diam}(G)$, is the maximum distance between any two vertices of G.

Cliques and independent sets

Definition: A **clique** K in a graph G is a subgraph of G which is isomorphic to a complete graph.

Definition: The **clique number** of a graph G, (denoted $\omega(G) =$ "omega") is size of the maximum clique in G.

Definition: An **independent set** in a graph G is a subset $X \subset V(G)$ such that no edge of G connects any two vertices of X. In other words, the induced subgraph of G on X contains no edges.

Definition: The **independence number** of a graph G, (denoted $\alpha(G) =$ "alpha") is the size of the maximum independence set of G.

Vertex covers

Definition: A **vertex cover** of a graph G is a subset $X \subset V(G)$ such that X contains (at least) one endpoint of every edge in G.

$\star$ Note: This set of vertices covers the *edges* of G. $\star$

Definition: The size of the minimum vertex cover is denoted $\beta(G)$.

Theorems: Let G be a graph and suppose $X \subset V(G)$.

1. X is a clique in $G \iff X$ is an independent set in G^c.
2. X is an independent set in $G \iff X^c$ is a vertex cover in G.
3. For all graphs G, $\alpha(G) + \beta(G) = |V(G)|$.

Recapitulation of graph statistics so far

$\delta(G)$ = minimum vertex degree

$\Delta(G)$ = maximum vertex degree

$\kappa(G)$ = (vertex) connectivity

$\lambda(G)$ = edge connectivity

$\mathrm{g}(G)$ = girth

$\mathrm{diam}(G)$ = diameter

$\omega(G)$ = clique number

$\alpha(G)$ = independence number

$\beta(G)$ = minimum vertex cover