Deterministic versus Probabilistic

Deterministic: All data is known beforehand

- Once you start the system, you know exactly what is going to happen.
- *Example.* Predicting amount of money in a bank account.
  - If you know the initial deposit, and the interest rate
  - You can determine the amount in the account after one year.

Probabilistic: Element of chance is involved

- You know the likelihood that something will happen, but don't know when.
- *Example.* Roll a die until it comes up '5'.
  - Know that in each roll, a '5' will come up with probability $1/6$.
  - Don't know exactly when, but we can predict well.

Basic Probability

Definition: An **experiment** is any process whose outcome is uncertain.

Definition: The set of all possible outcomes of an experiment is called the **sample space**, denoted X or S.

Definition: Each outcome $x \in X$ has a number between 0 and 1 that measures its likelihood of occurring. This is the **probability** of x, denoted $p(x)$.

Example. Rolling a die is an experiment; the sample space is $\{\underline{\hspace{3cm}}\}$. The individual probabilities are all $p(i) =$ ____.

Definition: An **event** E is something that happens (in other words, a subset of the sample space).

Definition: Given E, the **probability** of the event ($p(E)$) is the sum of the probabilities of the outcomes making up the event.

Example. The roll of the die ... [is '5'] or [is odd] or [is prime] ...

Example. $p(E_1) =$ ______, $p(E_2) =$ ______, $p(E_3) =$ ______.

Independent Events

Definition: Two events are **independent** if the probabilities of occurrence do not depend on one another.

Example. Roll a red die and a blue die.

- Event 1: blue die rolls a 1. Event 2: red die rolls a 6. These events are independent.
- Event 1: blue die rolls a 1. Event 2: blue die rolls a 6. These events are dependent.

Example. Pick a card, any card! Shuffle a deck of 52 cards.

- Event 1: Pick a first card. Event 2: Pick a second card. These events are ____________.

Example. You wake up and don't know what day it is.

- Event 1: Today is a weekday. E_1 vs. E_2
- Event 2: Today is cloudy. E_2 vs. E_3
- Event 3: Today is Modeling day. E_1 vs. E_3

Independent Events

- When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,

$$p(E_1 \text{ and } E_2) = p(E_1)p(E_2).$$

Example. What is the probability that today is a cloudy weekday?

- When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,

$$\begin{aligned} p(E_1 \text{ or } E_2) &= 1 - (1 - P(E_1))(1 - P(E_2)) \\ &= P(E_1) + P(E_2) - p(E_1)p(E_2) \end{aligned}$$

Proof: Venn diagram / rectangle

Example. What is the probability that you roll a blue 1 OR a red 6?

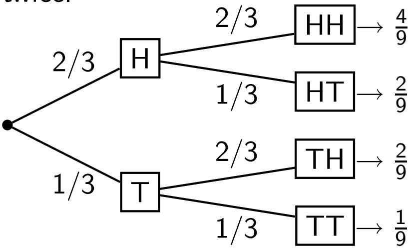
This does not work with *dependent* events.

Decision Trees

Definition: A **multistage** experiment is one in which each stage is a simpler experiment. They can be represented using a **tree diagram**.

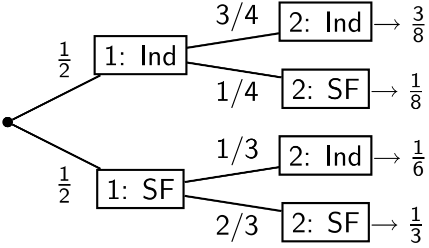
Each branch of the tree represents one outcome x of that level's experiment, and is labeled by $p(x)$.

Example. Flipping a biased coin twice.



Independent or dependent?

Example. Indiana and SF State U. play two soccer games. (p. 382)



Independent or dependent?

Expected value / mean

"Even with the randomness, what do you expect to happen?"

Suppose that each outcome in a sample space has a number $r(x)$ attached to it. (examples: number of pips on a die, amount of money you win on a bet, inches of precipitation falling)

This function r is called a **random variable**.

Definition: The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$\mu = E[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).$$

Idea: With probability $p(x_1)$, there is a contribution of $r(x_1)$, etc.

Example. How many heads would you expect on average when flipping a biased coin twice?

Example. How many wins do you expect Indiana to have?

Expected value / mean

When two random variables are on two independent experiments, the expected value operation behaves nicely:

$$E[X + Y] = E[X] + E[Y] \text{ and } E[XY] = E[X]E[Y].$$

Example. We throw a red die and a blue die. What is the expected value of the sum of the dice and the product of the dice?

$_b+^r$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$_b*^r$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$E[X + Y] =$

$E[XY] =$