Errors inherent to the modeling process

Models always have errors ⇝

► Be aware of them.
► Understand and account for them!
► Include in model discussion.

Types of Errors

1. **Formulation Errors** occur when simplifications or assumptions are made. (⋆)

2. **Observation Errors** occur during data collection. (⋆)

3. **Truncation Errors** occur when you approximate an incalculable function.

4. **Rounding Errors** occur during calculations when your computing device can’t keep track of exact numbers.
Errors inherent to the modeling process

1 **Formulation Errors** occur when simplifications or assumptions are made.

Example from the book, pp. 70–73: Seismology.

Set off an explosion at one place and measure it at another (dist. $D$). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive $T$, and the second shock $T'$.

$$d = \frac{D}{2} \sqrt{\left(\frac{T'}{T}\right)^2 - 1}$$

**Assumptions:** The earth is flat, and the layer is parallel to the surface. If layers are not parallel (off by $\alpha^\circ$), the percent errors can be large!

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>% error</td>
<td>3.4</td>
<td>18</td>
<td>37</td>
<td>105</td>
</tr>
</tbody>
</table>
Observation Errors occur during data collection.

Continuation of the previous example:

Even if the layers are parallel, perhaps our timing is inaccurate. Let’s say that $T$ is 1 second and $T'$ is 1.2 seconds, but that our timer is off by at most 1%.

Then $T$ might be ____ seconds or ____ seconds, and $T'$ might be ____ seconds or ____ seconds.

<table>
<thead>
<tr>
<th>$T$</th>
<th>over</th>
<th>over</th>
<th>under</th>
<th>under</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T'$</td>
<td>over</td>
<td>under</td>
<td>over</td>
<td>under</td>
</tr>
</tbody>
</table>

% error in $d$: -0.5%, -5%, +6%, 0%

One way to decrease influence: measure many times, take average.
Errors inherent to the modeling process

3 Truncation Errors occur when you approximate an incalculable function.

Question: When is \( x^5 + x - 1 = 0 \)? What is \( \sin 1 \)? Numerically?

Answer: Use a Taylor series approximation:
\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots.
\]

4 Rounding Errors occur during calculations when your computing device can’t keep track of exact numbers.

Question: What is \( 1.2300001^{10} \)?

Answer: If we only have three-digit accuracy, then
\[
1.23 \cdot 1.23 = 1.51, \quad 1.23 \cdot 1.51 = 1.86 \quad \ldots \quad 1.23^{10} = 7.95
\]
\[
1.2300001 \cdot 1.2300001 = 1.5129002, \\
1.2300001 \cdot 1.5129002 = 1.8608674, \\
1.2300001^{10} = 7.9259523
\]
True answer: 7.925952539912863452584748018737649320039805\ldots