Errors inherent to the modeling process

Models always have errors $\rightsquigarrow$

- Be aware of them.
- Understand and account for them!
- Include in model discussion.

Types of Errors

1. **Formulation Errors** occur when simplifications or assumptions are made. **($\star$)**
2. **Observation Errors** occur during data collection. **($\star$)**
3. **Truncation Errors** occur when you approximate an incalculable function.
4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

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1. **Formulation Errors** occur when simplifications or assumptions are made.

Example from the book, pp. 70–73: Seismology.

Set off an explosion at one place and measure it at another (dist. D). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive T, and the second shock T'.

$$d = \frac{D}{2}\sqrt{(T'/T)^2 - 1}$$

Assumptions: The earth is flat, and the layer is parallel to the surface.

If layers are not parallel (off by α°), the percent errors can be large!

α	1	5	10	30
% error	3.4	18	37	105

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2 **Observation Errors** occur during data collection.

Continuation of the previous example:

Even if the layers are parallel, perhaps our timing is inaccurate.
Let's say that T is 1 second and T' is 1.2 seconds,
but that our timer is off by at most 1%.

Then T might be ____ seconds or ____ seconds,
and T' might be ____ seconds or ____ seconds.

T	over	over	under	under
T'	over	under	over	under
% error in d	-0.5%	-5%	+6%	0%

One way to decrease influence: measure many times, take average.

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3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

Question: What is 1.2300001^{10}?

Answer: If we only have three-digit accuracy, then

$1.23 \cdot 1.23 = 1.51, \qquad 1.23 \cdot 1.51 = 1.86 \qquad \ldots \qquad 1.23^{10} = \mathbf{7.95}$

$1.2300001 \cdot 1.2300001 = 1.5129002,$

$1.2300001 \cdot 1.5129002 = 1.8608674,$

$1.2300001^{10} = \mathbf{7.9259523}$

True answer: $7.9259525399128634525847480187376493200398 05\cdots$