Data Fitting

Definition: A mathematical model is **descriptively realistic** if it is deduced from a believable description of the system being modeled.

Example. **Full moons.** A full moon appears to occur every 29 days. Let M_L, M_N be the dates of the last and next full moons. Is the model

$$M_N = M_L + 29$$

descriptively realistic? _____ Why?

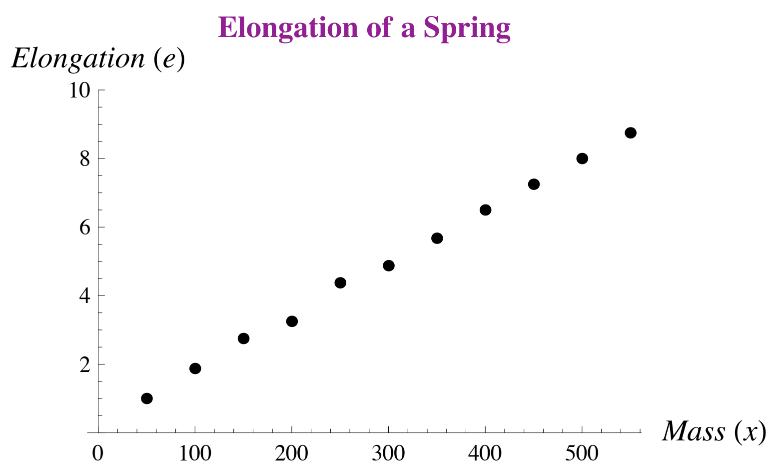
Given a descriptively realistic model that gives a set relationship that depends on a constant, how do we determine this constant?

Springs and Elongations

Example: Modeling Spring Elongation

Take your favorite spring. Attach different masses.
How much does it stretch from rest? [Its **elongation**.]

When we plot the data, we get the following **scatterplot**.



m	e
50	1.000
100	1.875
150	2.750
200	3.250
250	4.375
300	4.875
350	5.675
400	6.500
450	7.250
500	8.000
550	8.750

What do you notice? ______________________

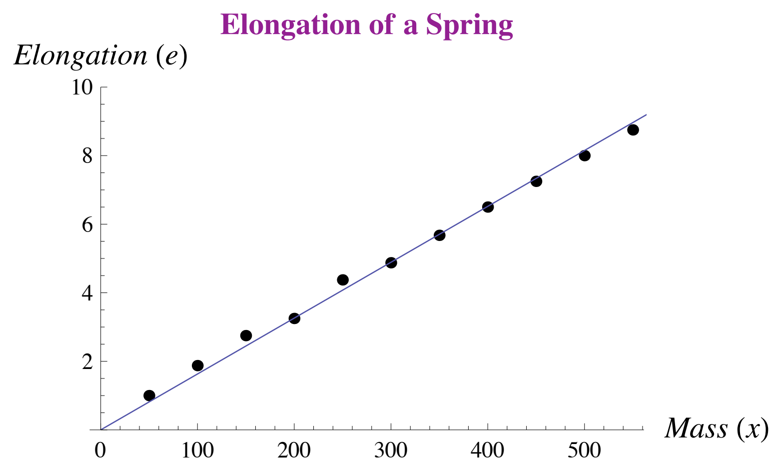
Proportionality

When data seems to lie on a line through the origin, we expect the two variables to be **proportional**; in this case, $e = km$ for some constant k.

We need to find this **constant of proportionality** k.

So: Estimate the slope of the line. **How?**

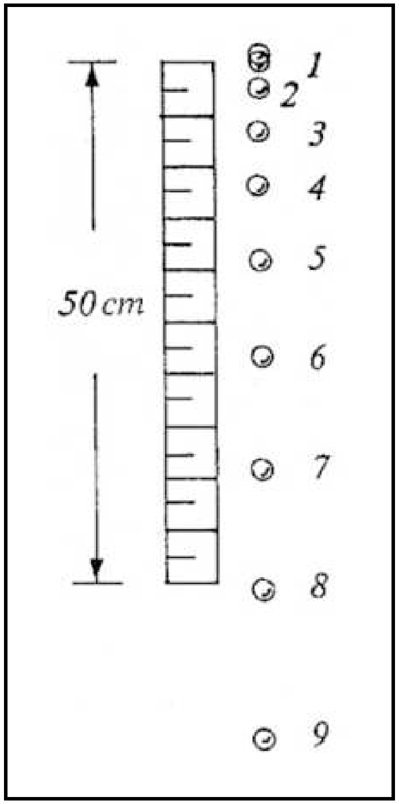
1. Guesstimating



2. Mathematically: **Linear Regression / Least Squares**
   (For another day)

Fitting Gravity Data

Example. Modeling the dropping of a golf ball

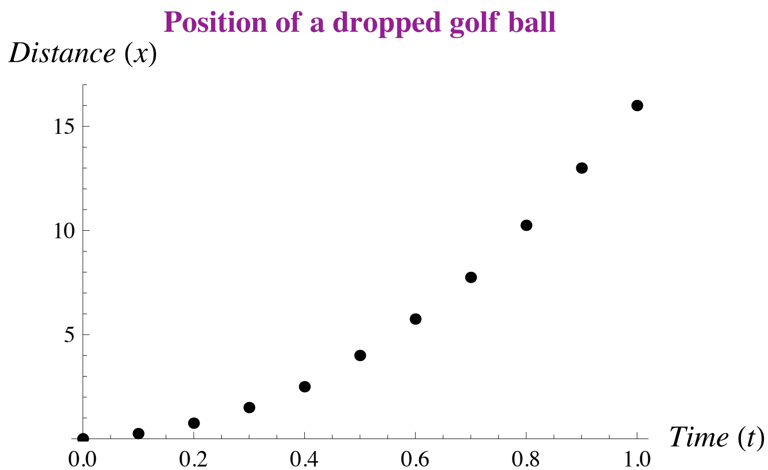


Source:
practicalphysics.org

Let's use an experiment to test the gravity model from last time.

Use a camera to record the position every tenth of a second.

Data would be similar to the table; plotted in the scatterplot below.



t	x
0.0	0.00
0.1	0.25
0.2	0.75
0.3	1.50
0.4	2.50
0.5	4.00
0.6	5.75
0.7	7.75
0.8	10.25
0.9	13.00
1.0	16.00

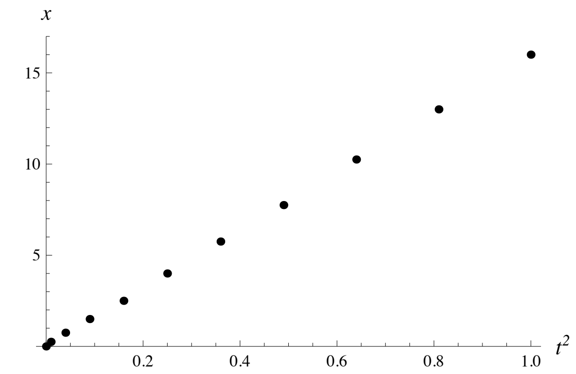
[Ignore data on p. 25.]

Fitting Gravity Data

These data seem to fit a $\underset{\text{(type of function)}}{\underline{\hspace{4cm}}}$. How can we be sure?

1. Plot distance as a function of t^2.

x as a function of t^2



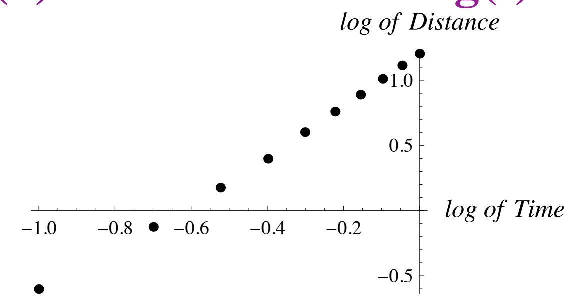
2. ⋆ Plot the log of distance as a function of log of time. ⋆

- Suppose $x = Ct^2$.
- Taking the *log* or *ln* of both sides gives

$$\log x = \log(Ct^2) =$$

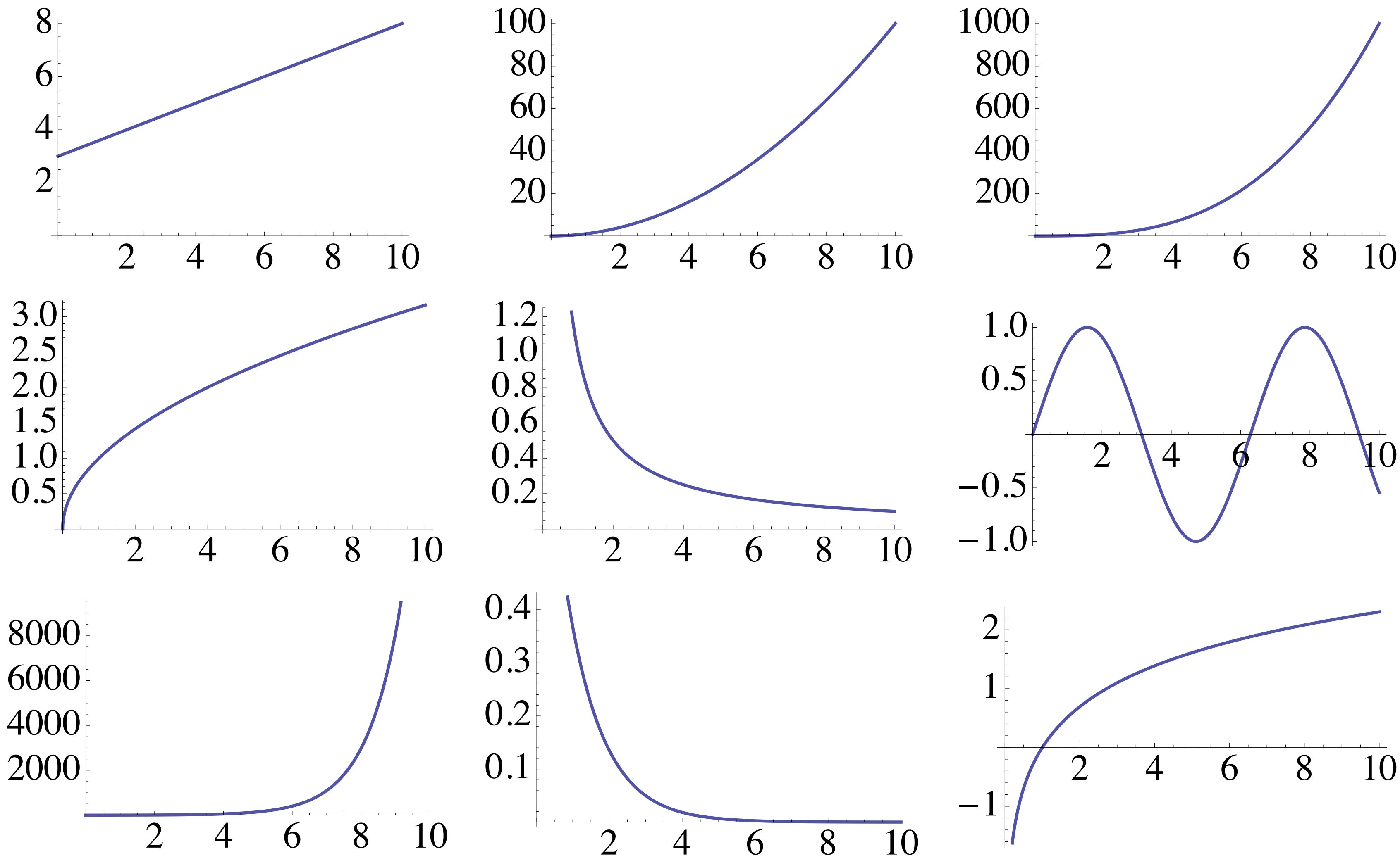
- So all you need to do is fit to a line and solve for C!

log(x) as a function of log(t)



$$\log x \approx 2 \log t + 1.2$$

Functions You Should Recognize on Sight

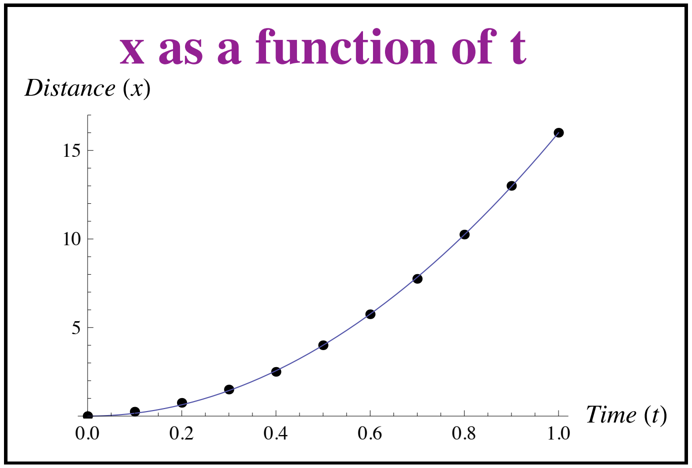


Fitting Gravity Data

We have determined that our gravity model

$$x(t) = 16t^2$$

appears to model the dropping of a golf ball.



Is our model descriptively realistic? _____
Why?

Example. Raindrops—Our model gives their position as $x(t) = 16t^2$.

A raindrop falling from 1024 feet would land after $t = 8$ seconds.

However, an experiment shows that the fastest drop takes 40 seconds, and that drops fall at different rates depending on their size.

Even if we have a good model for one situation doesn't mean it will apply everywhere. *We always need to question our assumptions.*

—Extensive discussion in Section 1.3.—