Group Worksheet, 3 May 2010

Carpenter's Problem: Let x represent the number of tables built and y represent the number of bookcases built. Profit (z) earned: \$25 per table and \$30 per bookcase. There are 690 units of lumber and 120 units of labor available. This translates into the following linear program (LP).

$$
\begin{array}{rll}
\text{(LP) maximize} & z = 25x + 30y & \text{(objective function)} \\
\\
\text{subject to} & 20x + 30y \leq 690 & \text{(lumber constraint)} \\
& 5x + 4y \leq 120 & \text{(labor constraint)} \\
& x, y \geq 0 & \text{(nonnegativity constraints)}
\end{array}
$$

1. Calculate the solution to the carpenter's problem by hand. (It will help to draw the feasible region.)

2. Now work to determine the increase in profit if the carpenter is able to obtain 760 units of lumber. From this calculation, determine the price of lumber per unit at which it would be advantageous to buy more lumber. [*Hint: Understand which aspect of the linear program is changed by the increase in profit. Modify your drawing of the feasible region to determine a new corner point. Determine the value of the objective function here.*]

3. We return to the original lumber constraint. Similarly to question 2, determine the increase in profit if we are able to obtain 134 units of labor. From this calculation, determine the hourly cost of unit labor at which it would be advantageous to hire more labor.

4. We return to the original constraints, and now we allow the profit per table to vary. Over what range of profit per table would we obtain the same optimal solution of $(12, 15)$? What happens outside that range of prices? [*Hint: This is changing the slope of the constant-objective line. What happens to the optimal value if the slope changes too much? What does "too much" mean?*]