Numerical Derivatives and Max/Min

The goal of this worksheet is to understand the commands in the <u>CALC</u> menu. These commands allow you to calculate numerically the zeroes, minima, and maxima of a function, and find a numerical approximation for the derivative of a function at an x-value.

Type in the function $y = x^4 + x^3 - 6x^2 - x + 6$. First <u>Zoom</u> Standard in order to understand what the function looks like. Then <u>Zoom</u> Decimal to zoom in near the roots of the function.

Here is what the <u>CALC</u> menu can help us learn:

- <u>VALUE</u>: This gives you the y-value that the function takes on at a particular x-coordinate. With a function in the <u>Y=</u> menu, push <u>CALC</u> <u>VALUE</u>. Type in an x-value, perhaps 2.2 and press <u>ENTER</u>. Then the function value at $x = 2.2$ is given. This is useful if tracing is not precise enough.

- <u>ZERO</u>: This command tells you approximately for which x-values the function $f(x) = 0$. This is useful when you are trying to solve MIN/MAX and CCU/CCD problems or any other problem in which you need to solve an inequality. Type <u>CALC</u> <u>ZERO</u>. The calculator will ask you for a range in which to test for a value—first the left bound, then the right bound— and then a guess for where the zero is. To input these values you can either trace along the function and press enter, or type in the numerical values using the keypad. Try to find the zeroes of the above polynomial. As a start, try left bound -3, right bound -2, and guess -2.8. The four roots of the polynomial are:

  - Root 1:
  - Root 2:
  - Root 3:
  - Root 4:

- MINIMUM/MAXIMUM: This command gives the approximate x- and y-values for local minima and local maxima. The method is similar to finding zeroes, above. Try to find the local extrema of the polynomial. For example, to find the local maximum, type in <u>CALC</u> <u>minimum</u> and then use a left bound of -1, a right bound of 1 and a guess of 0.

  - Coordinates of global minimum:
  - Coordinates of local maximum:
  - Coordinates of local minimum:

- <u>DY/DX</u>: This command gives a numerical approximation for the derivative of the function at an entered x-value. Type <u>CALC</u> <u>dy/dx</u>. Then either trace to an x-value or type in an x-value. The calculator will output the approximate value for dy/dx there.

Use your new graphing calculator skills to answer the following two questions:

1. Find the first and second derivatives of $f(x) = x^4 + x^3 - 6x^2 - x + 6$ by hand and input them as functions in the <u>Y=</u> menu. Determine where $f(x)$ is increasing, decreasing, concave up, and concave down.

2. Determine the maximum profit and minimum average cost when production of q units of jello costs $C(q) = (q^2 + 1)/(q + 3)$ dollars and can be sold for $p(q) = 1.0625 - 0.0025q$ dollars a piece.