

Generating Function Practice

Determine compact expressions for the following power series.

(1) $1 + x + x^2 + x^3 + x^4 + x^5$ (a polynomial)

(2) $2 + x + x^3 + x^5 + x^7 + x^9 + \dots$

(3) $1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots$

(4) $1 + x + x^2/2 + x^3/3 + x^4/4 + x^5/5 + \dots$

Find a formula for h_n if its generating function $H(x) = \sum_{n \geq 0} h_n x^n$ has the following compact form.

(1) $g(x) = 1/(1 + x^3)$

(2) $g(x) = e^{3x} - e^{-3x}$

(3) $g(x) = \arctan(x)$ [*Hint: what is the derivative of $\arctan(x)$?*]

(4) $g(x) = x/(1 - 2x)^2(1 + 5x)$