

MATH 636, Fall 2013

HOMEWORK 7

due 5:00PM on Thursday, November 21.

Background reading: Combinatorics: A Guided Tour, Sections 2.4, 3.4, and 4.4.
(Ignore any discussion of Exponential Generating Functions.)

Follow the posted homework guidelines when completing this assignment.

Please **only** consult with your classmates or professor to discuss the problem set.

These four questions are each worth five points each. Continue work on your project.

7-1. Exercise 4.4.2

7-2. (a) Determine the generating function for the number of partitions of n such that there are at most two parts of the same size.

[For example, 511 is OK, but 4111 is not allowed since 1 appears thrice.]

(b) Determine the generating function for the number of partitions of n such that the parts are all of size equal to a power of two.

[For example: 84422 is OK, but 744221 is not because 7 is not a power of two.]

7-3. This question is related to the number of standard Young tableaux of the partition $\lambda = (n - k) + 1 + 1 + 1 + \dots + 1$, which has one part of size $n - k$ and k parts of size 1. (This is a partition of the integer n .)

(a) Determine the number of standard Young tableaux of shape λ using the hook length formula.

(b) Determine the number of standard Young tableaux of shape λ without using the hook length formula—count them directly.

7-4. How many ways are there to take a line of n soldiers, break them into consecutive non-empty platoons, and choose some (possibly empty) subset of each platoon to be on “night watch”? Give an exact answer, not simply a generating function.