

MATH 636, Fall 2013

HOMEWORK 5

due 5:00PM on Thursday, October 31.

Background reading: Combinatorics: A Guided Tour, Sections 3.3 and 3.5, and pp. 114–115. (Ignore any discussion of Exponential Generating Functions.)

Follow the posted homework guidelines when completing this assignment.

Please **only** consult with your classmates or professor to discuss the problem set.

5-1. 3.3.2, parts (b), (d), (e), and (f).

5-2. Give a combinatorial counting question whose answer is the coefficient of x^k in the generating function

$$f(x) = \frac{(1+x)^n}{(1-x)^m}.$$

5-3. Prove that

$$\sum_{j=2}^{\infty} \frac{1}{\binom{j}{2}} = 2.$$

[Yes, the denominator is the binomial coefficient $\binom{j}{2}$.]

[Hint: Write the denominator as a polynomial of j , then use partial fractions.]

5-4. Suppose you roll **three** six-sided dice. Calculate the probability that the dice sum to fourteen. Now suppose you roll **five** six-sided dice. Calculate the probability the dice sum to fourteen. Which probability is higher?

Use generating functions to solve this problem. I would also suggest some computer algebra software (Mathematica, Sage, Maple, Wolfram Alpha, etc.).

5-5. 3.5.1(c) and 3.5.1(d).