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$n \backslash k$	0	1	2	3	4	5	6	7
0	1							
1	1	1						
2	1		1					
3	1			1				
4	1				1			
5	1					1		
6	1						1	
7	1							1

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Seq's in Pascal's triangle:

- $1, 2, 3, 4, 5, \dots$ $\binom{n}{1}$
  $(a_n = n)$
- $1, 3, 6, 10, 15, \dots$ $\binom{n}{2}$
  triangular
- $1, 4, 10, 20, 35, \dots$ $\binom{n}{3}$
  tetrahedral
- $1, 2, 6, 20, 70, \dots$ $\binom{2n}{n}$
  centr. binom.

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- $1, 2, 3, 4, 5, \ldots$ $\binom{n}{1}$ ($a_n = n$) A000027
- $1, 3, 6, 10, 15, \ldots$ $\binom{n}{2}$ triangular A000217
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- $1, 2, 6, 20, 70, \ldots$ $\binom{2n}{n}$ centr. binom. A000984

Online Encyclopedia of Integer Sequences:
http://oeis.org/

Binomial Theorem

Theorem 2.2.2. Let n be a positive integer. For all x and y,

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{n-1}xy^{n-1} + y^n.$$

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Rewrite in summation notation!

Determine the generic term $[\binom{n}{k}x\quad y\quad]$ and the bounds on k

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From the n factors $(x+y)$, you must choose a "y" exactly k times. Therefore, $\binom{n}{k}$ ways. We recover the desired equation. □