Pascal's triangle

Pascal's identity gives us the recurrence $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$.
With initial conditions we can calculate $\binom{n}{k}$ for all n and k.
$\binom{n}{0} = 1$ and $\binom{n}{n} = 1$ for all n.

$n \backslash k$	0	1	2	3	4	5	6	7
0	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	6	4	1			
5	1	5	10	10	5	1		
6	1	6	15	20	15	6	1	
7	1							1

Online Encyclopedia of Integer Sequences:
http://oeis.org/

Seq's in Pascal's triangle:

$1, 2, 3, 4, 5, \ldots$ $\binom{n}{1}$
$(a_n = n)$ A000027

$1, 3, 6, 10, 15, \ldots$ $\binom{n}{2}$
triangular A000217

$1, 4, 10, 20, 35, \ldots$ $\binom{n}{3}$
tetrahedral A000292

$1, 2, 6, 20, 70, \ldots$ $\binom{2n}{n}$
centr. binom. A000984

Binomial Theorem

Theorem 2.2.2. Let n be a positive integer. For all x and y,

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{n-1}xy^{n-1} + y^n.$$

Rewrite in summation notation!
Determine the generic term $[\binom{n}{k}x \quad y \quad]$ and the bounds on k

$$(x+y)^n = \sum$$

- The entries of Pascal's triangle are the coefficients of terms in the expansion of $(x+y)^n$.

Proof. In the expansion of $(x+y)(x+y)\cdots(x+y)$, in how many ways can a term have the form $x^{n-k}y^k$?

From the n factors $(x+y)$, you must choose a "y" exactly k times. Therefore, $\binom{n}{k}$ ways. We recover the desired equation. □