Vandermonde's Identity (p.117)

$$\binom{m+n}{k} = \sum_{j=0}^{k} \binom{m}{j} \binom{n}{k-j}$$

Combinatorial proof

Generating function proof

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Big candy g.f.: $B(x)$.

Small candy g.f.: $S(x)$.

Total g.f.: $B(x)S(x)$

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Big candy g.f.: $B(x) = (1+x)^{20} = \sum_{i=0}^{\infty} \binom{20}{i} x^i.$

Small candy g.f.: $S(x)$.

Total g.f.: $B(x)S(x)$

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Big candy g.f.: $B(x) = (1+x)^{20} = \sum_{i=0}^{\infty} \binom{20}{i} x^i$.

Small candy g.f.: $S(x) = \frac{1}{(1-x)^{40}} = \sum_{i=0}^{\infty} \left(\!\!\binom{40}{i}\!\!\right) x^i$.

Total g.f.: $B(x)S(x)$

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Big candy g.f.: $B(x) = (1+x)^{20} = \sum_{i=0}^{\infty} \binom{20}{i} x^i.$

Small candy g.f.: $S(x) = \frac{1}{(1-x)^{40}} = \sum_{i=0}^{\infty} \left(\!\binom{40}{i}\!\right) x^i.$

Total g.f.: $B(x)S(x) = \sum_{k=0}^{\infty} \left(\sum_{i=0}^{k} \binom{20}{i} \left(\!\binom{40}{k-i}\!\right) \right) x^k$

Multiplication example

Example. How many ways are there to fill a halloween bag with 30 candies, where you can have up to one of each of 20 big candy bars and you can have as many as you want of 40 different tiny candies?

Big candy g.f.: $B(x) = (1+x)^{20} = \sum_{i=0}^{\infty} \binom{20}{i} x^i.$

Small candy g.f.: $S(x) = \frac{1}{(1-x)^{40}} = \sum_{i=0}^{\infty} \left(\!\binom{40}{i}\!\right) x^i.$

Total g.f.: $B(x)S(x) = \sum_{k=0}^{\infty} \left(\sum_{i=0}^{k} \binom{20}{i} \left(\!\binom{40}{k-i}\!\right) \right) x^k$

Conclusion: $[x^{30}]B(x)S(x) = \sum_{i=0}^{30} \binom{20}{i} \left(\!\binom{40}{30-i}\!\right)$