

MATH 634, Spring 2013

HOMEWORK 8

to be turned in at 4:30PM on Monday, May 6.

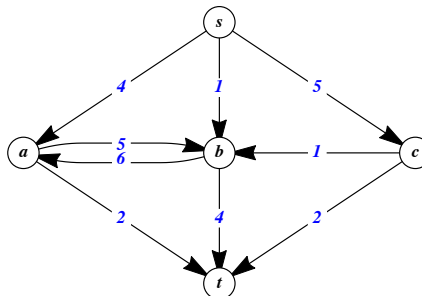
*Background reading: Pearls in Graph Theory, Section 7.2.*

- 8-1.** (a) Prove or disprove: For all  $n \geq 3$ , every maximal matching of  $W_n$  is a maximum matching. (b) Prove or disprove: For all  $n \geq 1$ , every maximal matching of  $K_{n,n}$  is a maximum matching.
- 8-2.** Run the Gale-Shapley algorithm twice on the following sets of preferences, once with the people proposing, and once with the pets proposing. Discuss how your results are related to the theorems on optimality and pessimality.

Pets' Preferences					
	Art	Bettie	Casper	Delphi	Ernie
1st Choice	F	F	G	G	H
2nd Choice	G	H	I	H	F
3rd Choice	I	I	F	J	I
4th Choice	J	G	H	I	G
5th Choice	H	J	J	F	J

People's Preferences					
	Fei	Georg	Helen	Ivan	Jin
1st Choice	C	E	A	E	B
2nd Choice	B	B	D	C	D
3rd Choice	E	C	B	A	A
4th Choice	A	A	E	B	C
5th Choice	D	D	C	D	E

- 8-3.** (a) List all  $st$ -cuts in the network pictured below (there are eight). Find the capacity of each  $st$ -cut and determine the min cut from this information.
- (b) Find a maximum flow for the network (and verify that it is a max flow). Then verify that the max flow / min cut theorem holds.  
*[Do not use the Ford-Fulkerson algorithm in this problem.]*



- 8-4.** Use the Ford-Fulkerson algorithm to find the max flow and min cut in the network below. You must prove that your flow is a max flow and your  $st$ -cut is a min cut.

