MATH 634, Spring 2013
Homework 7
to be prepared for presentation at 4:30pm on Monday, April 22.

Background reading: Pearls in Graph Theory, Sections 9.2, 10.3, 7.1, and 7.2.

7-1. This question has to do with planar duality:

(a) Show that for all n, the wheel graph W_n is self-dual.

(b) Find a graph that has two non-isomorphic planar duals.
[Hint: Look for different planar embeddings.]

7-2. Try to prove the Four Color Theorem by emulating the argument from class using Kempe Chains. What goes wrong?

7-3. 9.1.1ab

7-4. (a) Show that $\theta(K_7) = 2$.

(b) Show that genus$(K_6) = 2$

7-5. Use the Hungarian algorithm to solve problem 7.2.2.
Important: Use the initial matching $M = \{(a,4),(c,6),(e,2),(h,5)\}$.

7-6. A spanning tree T in a connected graph G is a subgraph of G that is a tree and includes every vertex of G. (See also p. 20 and Section 7.1 in the book.)

Prove the correctness of the following algorithm to find a spanning tree of a graph.

[Prove that the algorithm terminates and that the output of the algorithm is a spanning tree of G.]

Input: A connected graph G with n vertices.

Preprocess: Label the vertices 1 through n, color them all white. Let T be a set of edges, initially empty.

Repeat: For the lowest numbered white vertex v, order the edges incident with v. Going through each edge e from first to last, determine if including e in T would create a cycle in T. If it would not create a cycle, place e into T (T is growing.) If it would create a cycle, do not add e to T; go on to the next edge. Once every incident edge to v has been checked, color v black. If all vertices are black, go on to the next step. Otherwise, repeat this step.

Output: Output the graph T, a spanning tree of G.