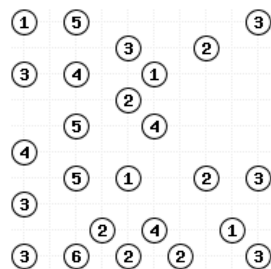


MATH 634, Spring 2013  
 HOMEWORK 4  
 due 4:30PM on Wednesday, March 20.

*Background reading: Pearls in Graph Theory, Section 3.1.*

- 4-1. (a) Prove that there is no closed knight's tour on the  $3 \times 8$  grid.  
 (b) Find a closed knight's tour on the  $3 \times 5$  **torus**.
- 4-2. (a) Find a graph  $E$  which has an Eulerian circuit but no Hamiltonian cycle.  
 (b) Find a graph  $H$  which has a Hamiltonian cycle but no Eulerian circuit.  
 [If either is impossible, prove why you can not find such a graph.]
- 4-3. Draw the binary de Bruijn graph of order  $n = 5$ . Find one binary de Bruijn sequence of order 5.  
 [Note: The graph will have 16 vertices and the sequence will be of length 32.]
- 4-4. Find a decomposition of the Grötzsch graph into the minimal possible number of paths.
- 4-5. Find a one-way Eulerian trail in the graph of Figure 3.3.6. (p. 67)  
 [Note: A discussion of one-way Eulerian trails in infinite graphs occurs above the figure.]
- 4-6. Sudoku is sooo last decade! Solve this Hashi puzzle.



**Instructions:** Draw in lines to connect the circles such that:

- Lines must be either perfectly vertical or horizontal.
- Up to two lines may be drawn connecting the same circles.
- The lines may not cross.
- The degree of each vertex is the enclosed number.
- The entire graph must be connected.

For many more Hashi puzzles and other fun games, visit  
<http://www.menneske.no/hash/eng/> & <http://www.puzzle-bridges.com/>.