

MATH 634, Spring 2013  
HOMEWORK 3  
due 4:30PM on Monday, February 25.

*Background reading: Pearls in Graph Theory, Sections 1.3 and 2.1.*

**3-1.** (a) Prove that if  $n$  is large enough, then the following statement is true:

For all graphs on  $n$  vertices, either  $G$  or  $G^c$  contains a cycle.

(b) For which  $n$  does this start to be true?

**3-2.** Prove Theorem 1.3.4, that every tree with at least one edge has at least two leaves.

**3-3.** Consider a tree  $T$  that has only vertices of degree 1, 2, and 3. Suppose that  $T$  has exactly 10 vertices of degree 3. Find and prove how many leaves  $T$  has.

*[Important: Prove your answer for **any** tree  $T$  satisfying these conditions.]*

**3-4.** Is Figure 2.1.5 critical? Justify.

*[Don't believe everything you read!]*

**3-5.** Let  $G$  be a graph with  $n$  vertices and  $n$  edges.

(a) Suppose  $G$  is connected. How many cycles does  $G$  have? Prove it.

(b) Suppose  $G$  is **NOT** connected. What can you say about the number of cycles in the graph? Can you determine a formula?

*[Part (b) is an exploration question. I want you to explore what happens and write up as much as you can about what you learn.]*