# MATH 634, Spring 2013
# Homework 2
## due 4:30pm on Wednesday, February 6.

*Background reading: Pearls in Graph Theory*, Sections 1.1 and 1.2.

**2-1.** Are any of these degree sequences graphic?

(a) 5 5 4 4 3 2 2 (b) 6 6 4 4 4 2 2 (c) 6 6 6 6 6 6 (d) 6 6 6 6 6 6 6

If you determine that the sequence is graphic, draw a graph with the given degree sequence. If you determine that the sequence is not graphic, prove it.

**2-2.** Prove that no graph has all degrees different. That is, prove that in a degree sequence of a graph, there is at least one repeated number.

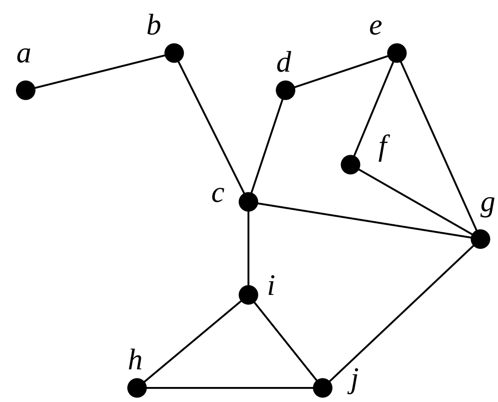
**2-3.** Explore the proof of Theorem 1.1.2.

The graph below has degree sequence ($\mathcal{S}_1$) 4 4 3 3 3 2 2 2 2 1. Define ($\mathcal{S}_2$) to be 3 2 2 2 2 2 2 2 1. Walk through the steps of the proof of Theorem 1.1.2 in the following way.

First, let us choose vertex $c$ from the graph to be vertex $S$ from the proof. Next, assign to each of the remaining vertices ($a - j$) a name of the form $T_i$ or $D_i$, just as in the proof.

(a) If you delete vertex $S$, does the new graph have degree sequence ($\mathcal{S}_2$)?

(b) Use the method in the proof to modify the original graph (possibly applying the algorithm multiple times) so that the resulting graph is such that removing $S$ gives a graph with degree sequence ($\mathcal{S}_2$).




**2-4.** Let $G$ be a connected regular graph with 22 edges. What are the possible number of vertices that $G$ may have?

**2-5.** Consider the graphs in Figure 1.2.4. Are any two of them isomorphic? Prove that you are correct.

**2-6.** Draw the Schlegel diagram for two of the following polyhedra: Icosidodecahedron, Truncated Icosahedron, Rhombicuboctahedron, Permutohedron of order 4, Snub Cube. (You will have to do some investigating to determine what these polyhedra are.)