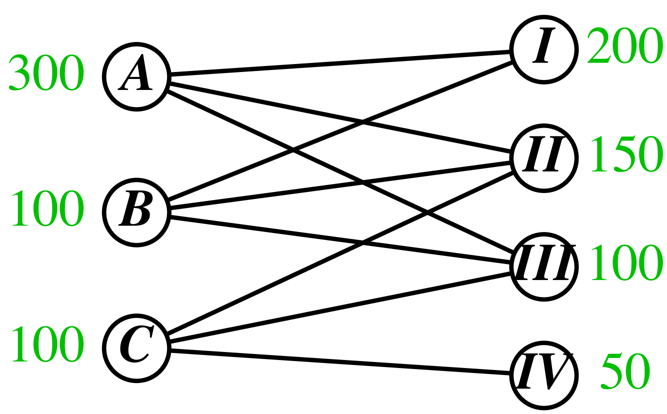
# Transshipment

**The Transshipment Problem:** Given $m$ suppliers and $n$ customers,

Is it possible for the customers (suppliers) to have their orders filled?

- Each supplier has some amount of product.
- Each customer desires some amount of product.
- Not all suppliers deliver to each customer.

Example. Suppliers $A$, $B$, $C$ have 300, 100, 100 units of product. Customers $I$, $II$, $III$, $IV$, desire 200, 150, 100, 50 units of product. Neither $A$ nor $B$ delivers to $IV$, and $C$ does not deliver to $I$.



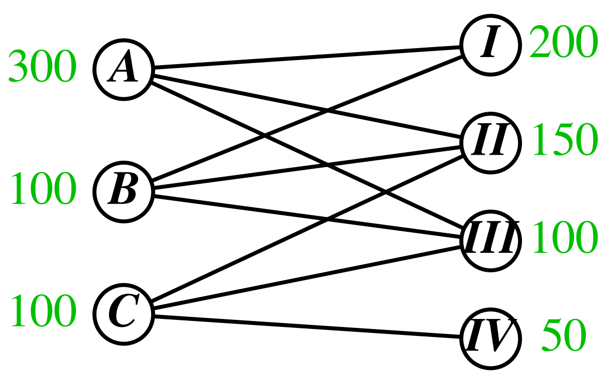

Q: Is there a transshipment that satisfies all the suppliers?

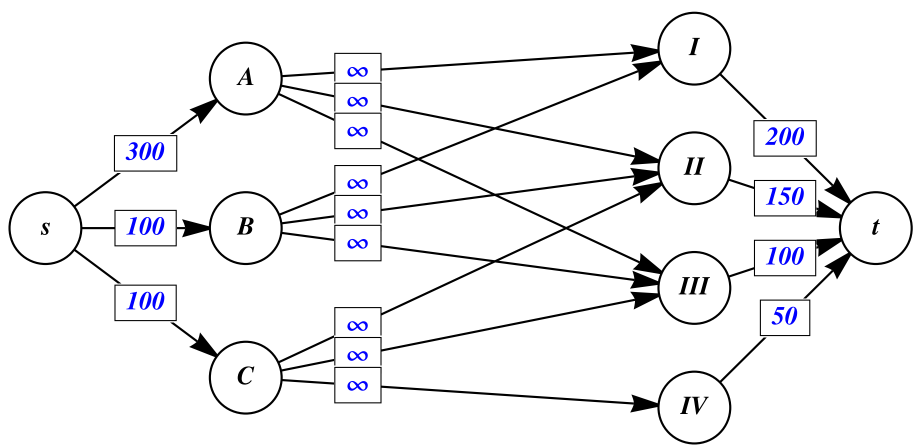
# Transshipment

Key: Convert the transshipment problem to a network flow problem.

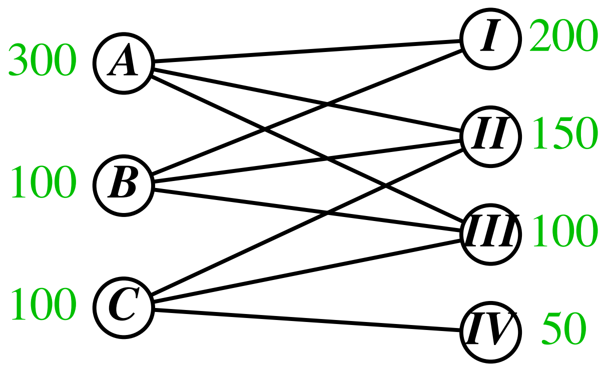
- Start with $G$, with edges **from** suppliers $x$ **to** customers $y$.
- Create a network $\widehat{G}$ by adding two vertices:
  - A "super-source" $s$ that is **adjacent to** every supplier $x$.
  - A "super-sink" $t$ that is **adjacent from** every customer $y$.
- Assign capacities to the edges as follows:

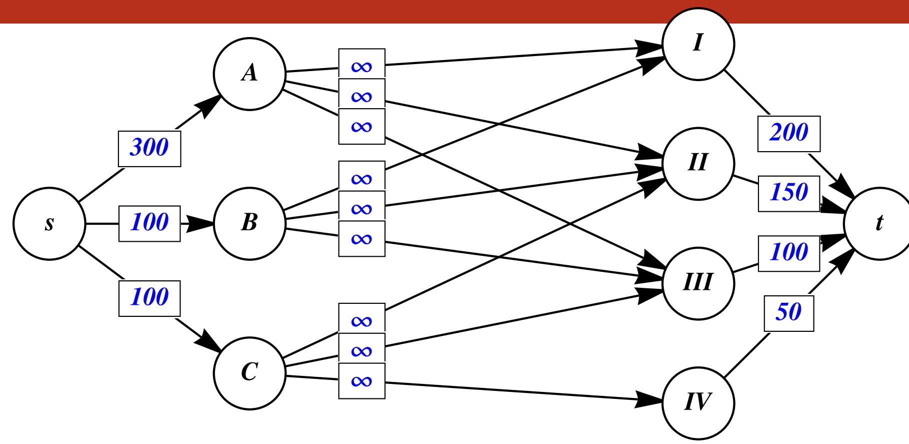
$$\begin{cases} \text{if } e : s \to x, \text{ set} & c_e = \text{supplier } x\text{'s supply} \\ \text{if } e : x \to y, \text{ set} & c_e = \infty \\ \text{if } e : y \to t, \text{ set} & c_e = \text{customer } y\text{'s demand} \end{cases}$$



# Transshipment




**Important:** a transshipment in $G \iff$ a flow in $\widehat{G}$.

$\therefore$ a maximum transshipment in $G \iff$ a maximum flow in $\widehat{G}$.

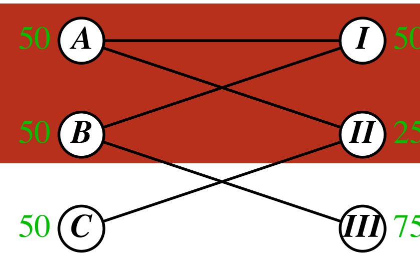
Run the Ford-Fulkerson algorithm. **Interpret the min cut.**

- When all suppliers are satisfied in $G$, the min cut in $\widehat{G}$ is ______.
- Otherwise, the min cut tells the problem: there exists a set of suppliers whose customers demand less than the suppliers supply.

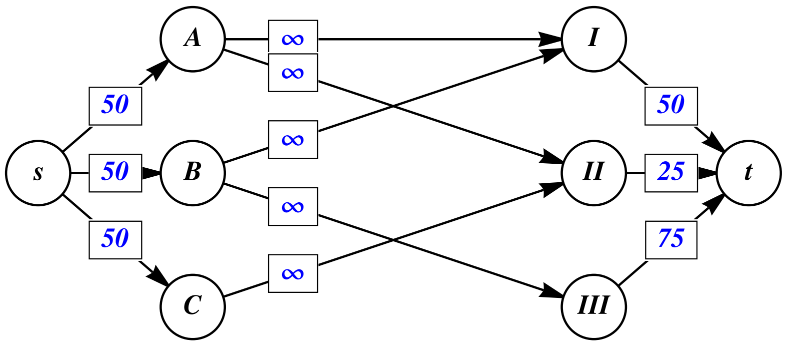
If you are customer-centric, orient the edges from right to left.

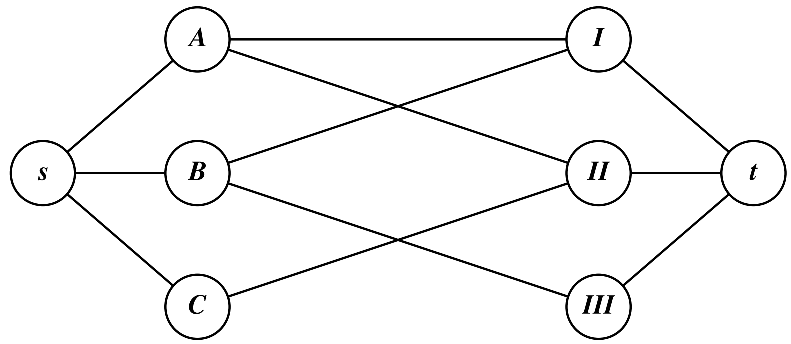
Gives a set of customers who can not be satisfied by their suppliers.

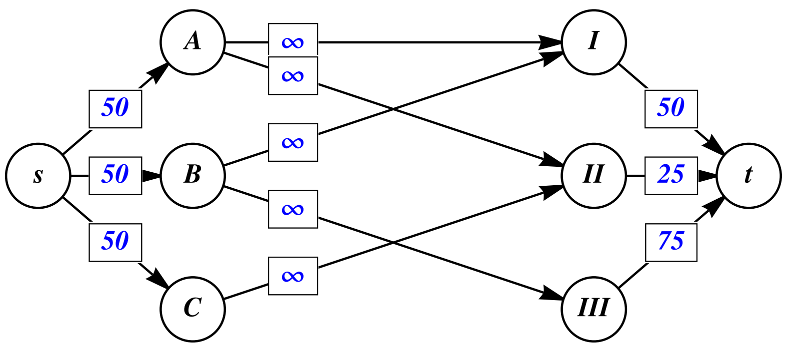
# Transshipment Example

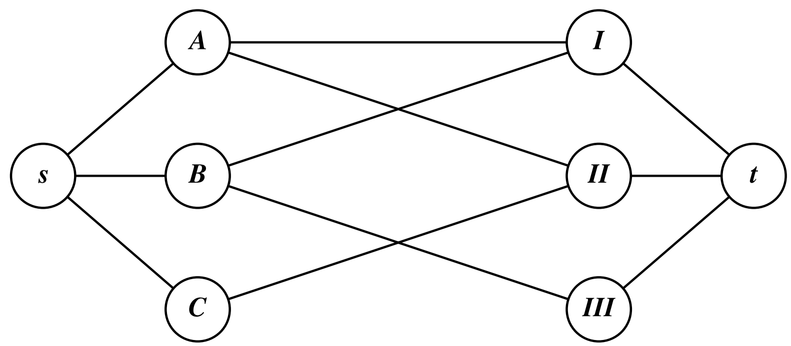



## Supplier-centric:



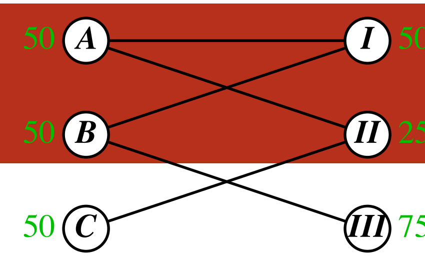



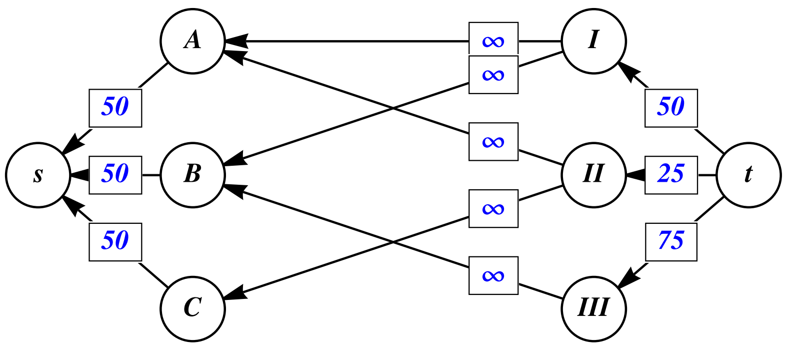



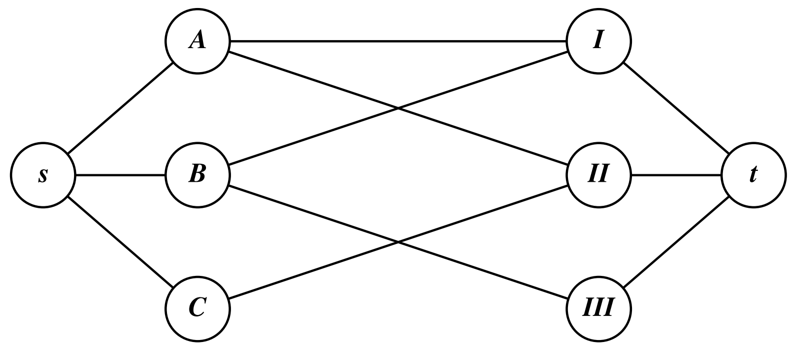

Problem:

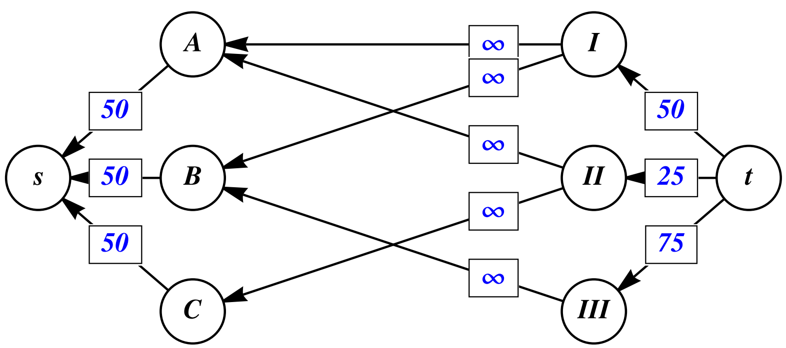
# Transshipment Example

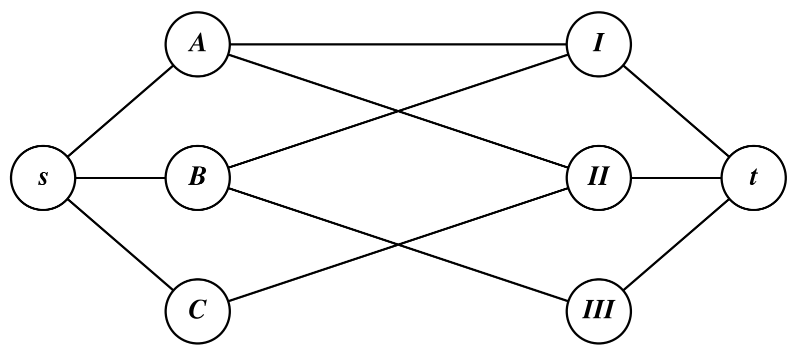



## Customer-centric:



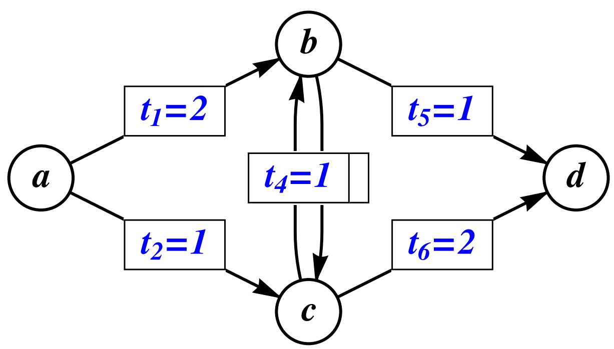

Problem:

# Dynamic Networks

- Ford–Fulkerson gives the max throughput of a static network.
- Use dynamic networks to model the act of sending shipments.

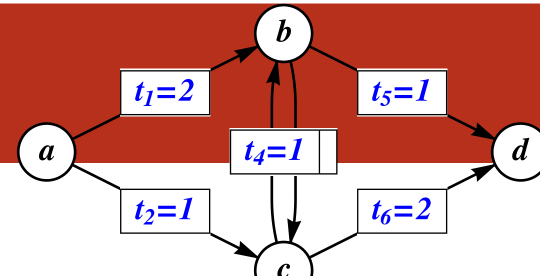
*Definition:* In a **dynamic network**, every edge $e$ has both a capacity $c_e$ and a travel time $t_e$.

Example. Consider four cities with warehouses ($a$, $b$, $c$, and $d$) such that one truck per day can leave along any route, and the travel time for each route is given by:




We wish to determine the maximum number of shipments which can make it from city $a$ on day 0 and arrive at city $d$ by day 5.

# Dynamic Networks




Create a new, static network.

- Create a vertex $v_i$ for every warehouse $v$ and every time $i$.
- For all original edges $e : v \to w$ with capacity $c_e$ and time $t_e$, create edges from $v_i \to w_{i+t_e}$ with capacity $c_e$ for all $i$.
- For all $v$ and $i$, create an edge from $v_i$ to $v_{i+1}$ with $\infty$ capacity. This represents shipping no product.
- Find the max flow from source(s) at time 0 to sink(s) at time $n$.

Example. In the graph below, calculate the max flow from $a_0$ to $d_5$.