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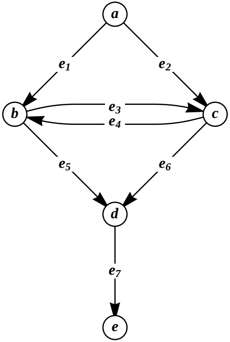

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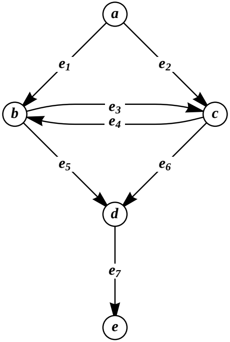

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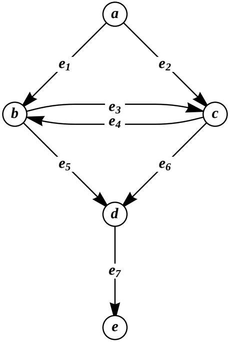

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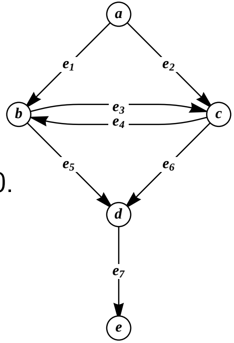

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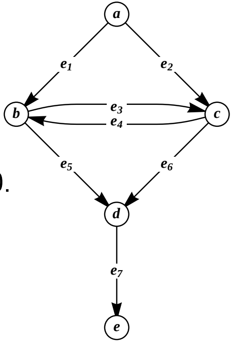

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Important. Any **path** or **cycle** in a digraph must respect the direction on each edge.

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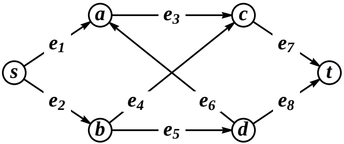

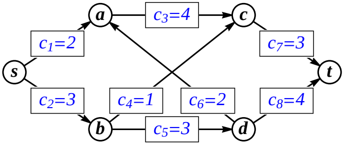

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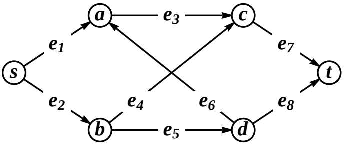

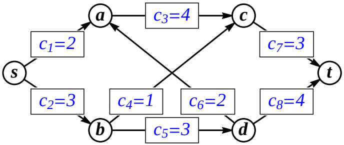

Idea. Graph networks represent real-world networks such as traffic, water, communication, etc.

Goal: Send as much "stuff" from s to t while respecting capacities.

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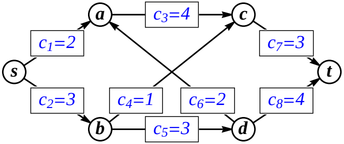

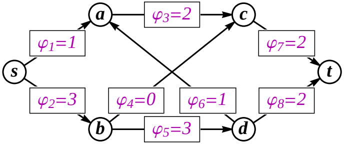

Definition: When $\varphi_e = c_e$, we say that e is **saturated**, or **at capacity**.

Maximum Flow

Theorem. Given a flow $\vec{\varphi}$ on a network G, the net flow out of s is equal to the net flow into t. Symbolically, $\displaystyle\sum_{e \text{ out of } s} \varphi_e = \sum_{e \text{ into } t} \varphi_e$.

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In G', flow is now conserved at every vertex except possibly t.
By Kirchhoff's Global Current Law (Theorem 6.2.2),
flow must be conserved at t as well.

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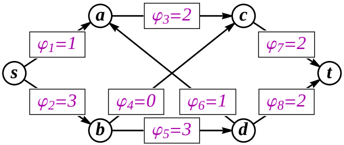
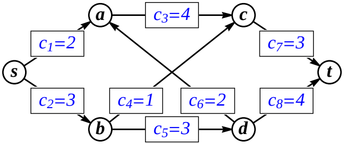
Definition: The **throughput** or **value** of a flow $\vec{\varphi}$ is $\sum_{e \text{ out of } s} \varphi_e$, denoted $|\vec{\varphi}|$.

Idea: The throughput is the amount of "stuff" flowing through G.

In our example, $|\vec{\varphi}| =$ ________________.





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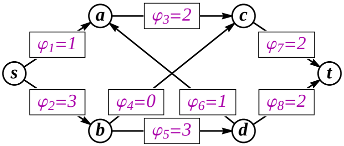

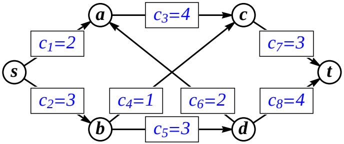

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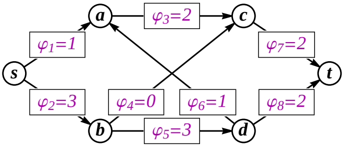

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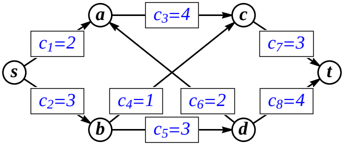

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$$\text{MAX FLOW} \qquad \underset{\text{over all flows } \vec{\varphi} \text{ on } G}{\text{maximize}} \quad |\vec{\varphi}|$$

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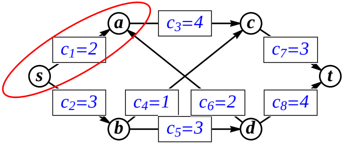

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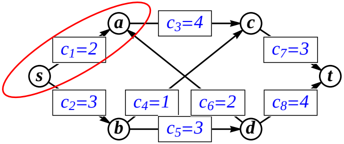

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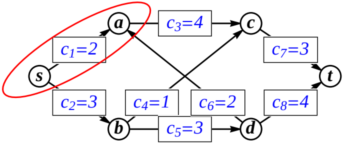

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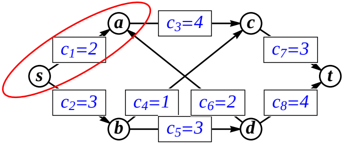

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So, if there exists a flow $\vec{\varphi}$ and st-cut $[X^*, X^{*c}]$ where equality holds, then $\vec{\varphi}$ is a maximum flow and $[X^*, X^{*c}]$ is a minimum cut

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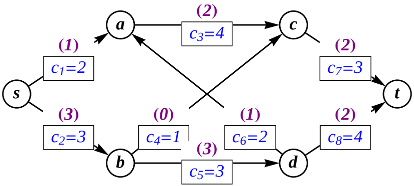

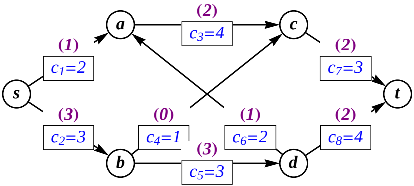

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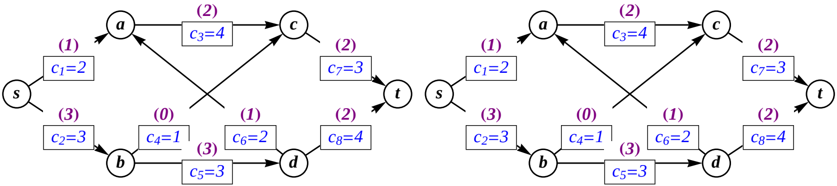

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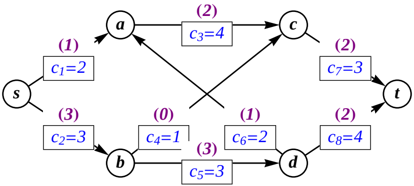

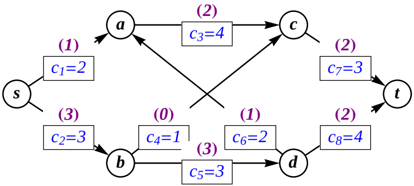

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We'll create a *companion graph* to keep track of augmenting paths.

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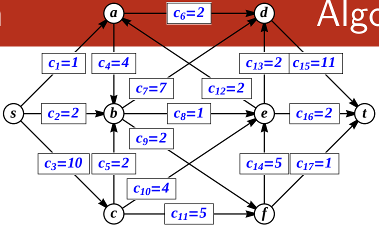
   → Upon STOP, the current flow is a maximum flow. ←
   In addition, let X be the set of vertices reachable from s in the flow companion graph. Then $[X, X^c]$ is a minimum st-cut.

A Ford–Fulkerson Algorithm Example



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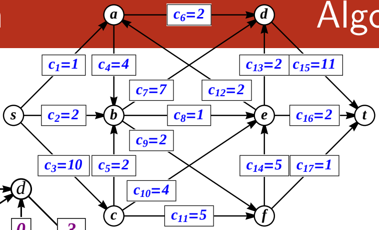

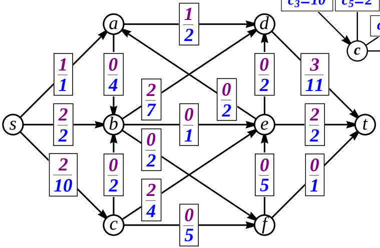

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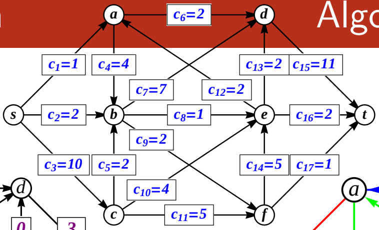
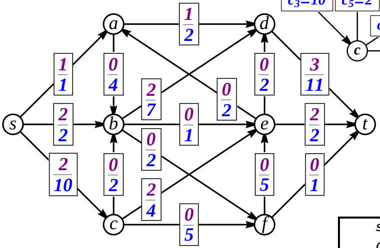
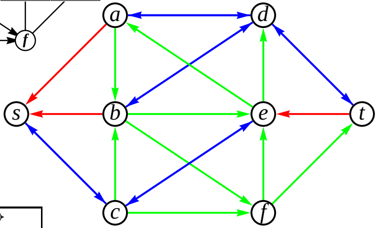




$s \to c \to b \to d \to t$: 2 units
$s \to c \to f \to e \to d \to t$: 2 units



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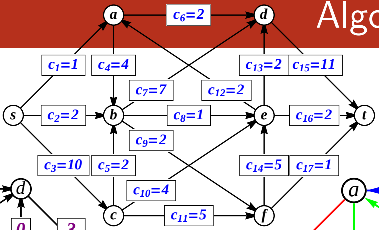

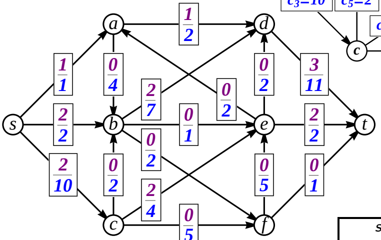

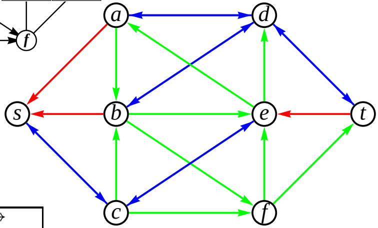

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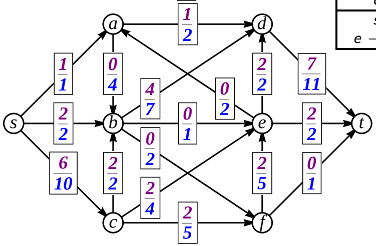

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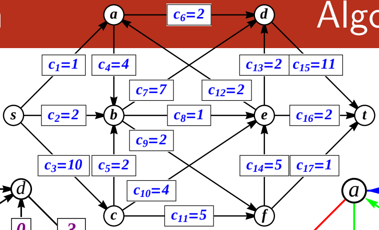

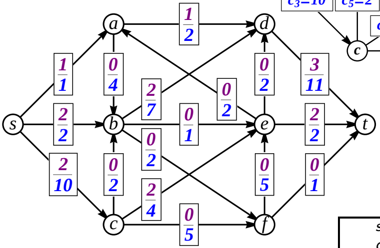

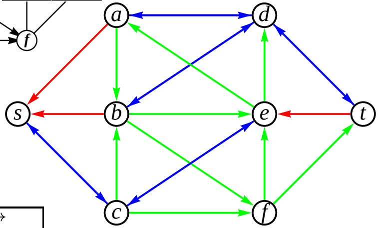

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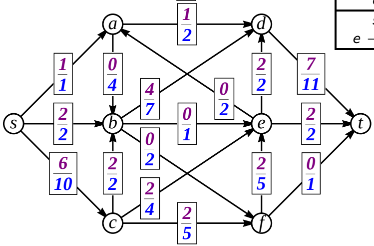

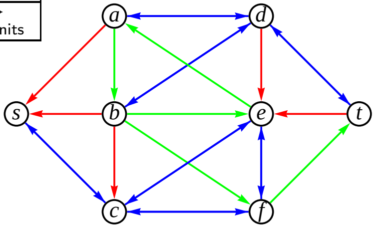

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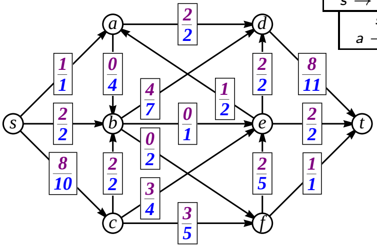

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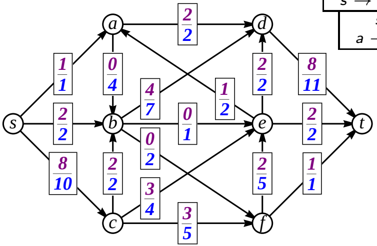

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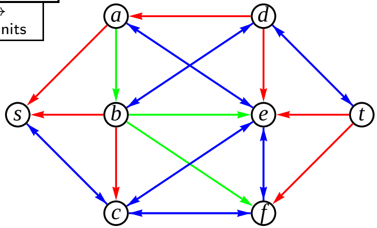

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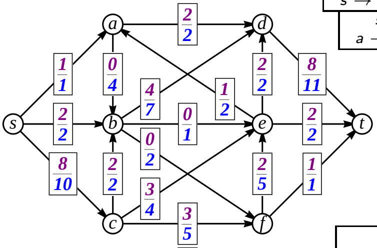

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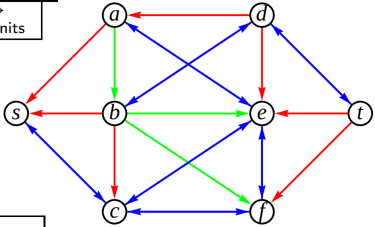

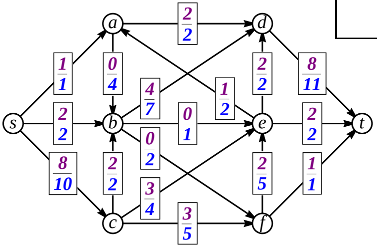

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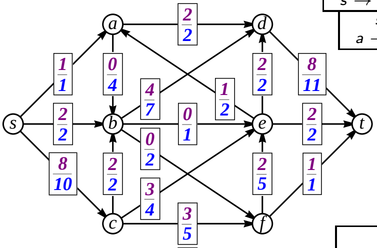

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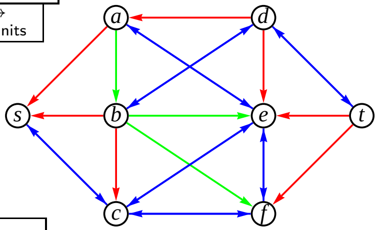

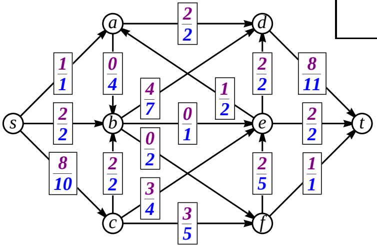

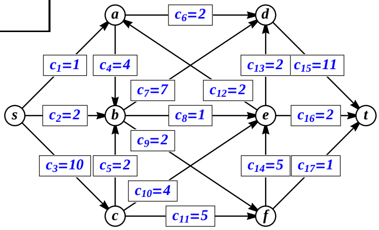

$$X = \{\_\_\_\_\_\_\_\}, \ [X, X^c] = \{\_\_\_\_\_\_\_\_\_\_\}, \text{ and } \big|[X, X^c]\big| = \_\_\_\_.$$

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Conclusion. The flow is a max flow and the st-cut is a min cut.

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- As presented here, this algorithm may be very slow.