MATH 245, Spring 2013
Homework 4
due 10:50am on Monday, April 8.

Background reading: Sections 1.5, 5.1, and 5.3A. You may also wish to read this webpage about Markov chains: http://www.sosmath.com/matrix/markov/markov.html

Follow the posted homework guidelines when completing this assignment.

4-1. (6 pts) Suppose you decide to visit Las Vegas (or Atlantic City, your choice) with exactly \$255 to spend. You decide to spend your time at the roulette wheel and bet either on red or black. As you may or may not know, a single-zero roulette wheel has eighteen red number spaces, eighteen black number spaces, and one green zero space, each as likely as one another. When you bet on red (or black), you win the amount of your bet if the ball lands on one of the red (or black) spaces, and lose the amount of your bet if not. Note that because of the green space, you will lose money on average.

(a) What is the expected value of your winnings by betting one dollar on red?
One strategy for spending all your money is to bet one dollar each of 255 times.
How much money would you expect to have at the end of the day?
[*Be explicit about what you are assuming when you give your answer.*]

(b) Now consider this new strategy:

- In your first game, you bet one dollar on red. If you win, you stop playing for the day, and walk away having won a net one dollar.
- If you lose, you play a second game but this time bet two dollars on red. Again, if you win, you walk away having won one dollar.
- If you lose, you play again and this time bet four dollars. Continue in this fashion, walking away if you win at any step, or betting twice as much in each game following a loss, until you have no more money.

Determine the expected value of your winnings by playing this strategy.
How much money do you expect to have at the end of the day?

(c) Do you prefer either of the strategies from part (a) or part (b), or would you spend your money in a different way?

4-2. (7 pts) Suppose that you are setting up a pizza delivery business with three stores, Alpha Pizza in Flushing, Beta Pizza in Long Island City, and Gamma Pizza in Jamaica.

When a customer calls a store, that store sends out a delivery person, who delivers the pizza and then returns to the closest store. Because of this, the delivery people end up transitioning from store to store throughout the night.

— Suppose that when Alpha Pizza is called, then 1/2 of the time the delivery person returns to Alpha, 1/5 of the time the delivery person goes to Beta, and 3/10 of the time goes to Gamma.

— When Beta Pizza is called, then 3/5 of the time the delivery person returns to Beta and with probability 1/5 the person goes to each of Alpha or Gamma.

— When Gamma Pizza is called, then 3/5 of the time the delivery person returns to Gamma, 4/15 of the time goes to Alpha, and 2/15 of the time goes to Beta.

(a) Set up a Markov Chain model to simulate this situation.

(b) Suppose that one evening there are 5 delivery people at each store at the beginning of the evening and they are all sent out at the same time. What is the expected distribution of the delivery people when they return from this first delivery?

(c) Determine the equilibrium distribution of the delivery people at the end of the day.

4-3. (7 pts) This question involves random simulation in *Mathematica*. As always, make sure to include text cells in your *Mathematica* notebook in order to explain what you are doing.

(a) Use the ideas involved in page 90 of the notes to create a simulation that flips five coins simultaneously and counts how many tails appear.

(b) Now use a `Table` command to repeat this experiment 1000 times. (Or more if you get carried away!) The result will be a list of 1000 numbers, each representing how many tails appear out of five. Take the average of this list by using the command `Mean`. Is the average what you expect?

(c) Input the list from part (b) into the `Histogram` command to see a visualization of the 1000 trials, and discuss how this is related to real-life coin flipping.

(d) Using basic probability, calculate the theoretical probability that when five coins are flipped, three turn up tails.

(e) Use the `Tally` command on the list from part (b) and see how often three of the five coins are tails in your simulation. How close is this to the theoretical answer?

4-4. Continue to work on your group project.