

Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Optimization: Inventory Policy

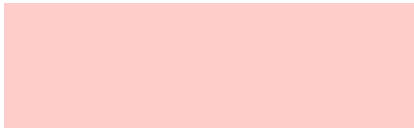
As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls.
Can adapt to market conditions.

Cons: Pay for delivery each day.



Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

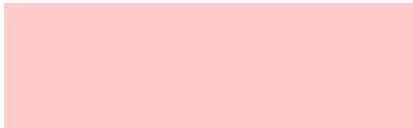
Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls.
Can adapt to market conditions.

Cons: Pay for delivery each day.

Suppose the delivery cost is \$100 per shipment.



Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls.
Can adapt to market conditions.

Cons: Pay for delivery each day.

Perhaps a year's worth?

Pros: Only pay delivery once!

Cons: Must store the balls.

Suppose the delivery cost is \$100 per shipment.

Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls.
Can adapt to market conditions.

Cons: Pay for delivery each day.

Suppose the delivery cost is \$100 per shipment.

Perhaps a year's worth?

Pros: Only pay delivery once!
Cons: Must store the balls.

Suppose the carrying cost is \$0.05 per ball per day.

Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls.
Can adapt to market conditions.

Cons: Pay for delivery each day.

Suppose the delivery cost is \$100 per shipment.

Perhaps a year's worth?

Pros: Only pay delivery once!
Cons: Must store the balls.

Suppose the carrying cost is \$0.05 per ball per day.

★ We are trying to find the optimal ordering schedule.

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3
4	×	\$0	40	\$2
5	×	\$0	20	\$1

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3
4	×	\$0	40	\$2
5	×	\$0	20	\$1
6	✓	\$100	100	\$5

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3
4	×	\$0	40	\$2
5	×	\$0	20	\$1
6	✓	\$100	100	\$5

Total delivery cost for 5 days: \$100

Total carrying cost for 5 days: $\$5 + \$4 + \$3 + \$2 + \$1 = \15 .

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3
4	×	\$0	40	\$2
5	×	\$0	20	\$1
6	✓	\$100	100	\$5

Total delivery cost for 5 days: \$100

Total carrying cost for 5 days: $\$5 + \$4 + \$3 + \$2 + \$1 = \15 .

How many deliveries in a year?

Total yearly cost:

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k - 1) + \dots + \$2 + \$1 =$

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k - 1) + \dots + \$2 + \$1 =$

How many deliveries in a year?

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k - 1) + \dots + \$2 + \$1 =$

How many deliveries in a year?

Total yearly cost: $C = \frac{365}{k} \left(100 + \frac{k(k+1)}{2} \right)$.

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k-1) + \dots + \$2 + \$1 =$

How many deliveries in a year?

Total yearly cost: $C = \frac{365}{k} \left(100 + \frac{k(k+1)}{2} \right)$.

Find the k that minimizes this function.

Solving $\frac{dC}{dk} = 365 \left(-\frac{100}{k^2} + \frac{1}{2} \right) = 0$

Gives $k \approx 14.1$. Answer?

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k-1) + \dots + \$2 + \$1 =$

How many deliveries in a year?

Total yearly cost: $C = \frac{365}{k} \left(100 + \frac{k(k+1)}{2} \right)$.

Find the k that minimizes this function.

Solving $\frac{dC}{dk} = 365 \left(-\frac{100}{k^2} + \frac{1}{2} \right) = 0$

Gives $k \approx 14.1$. Answer?

- ▶ There may be other considerations, such as a maximum or minimum shipment...

The language of optimization

Optimization questions cover a wide variety of situations.

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

Fact: You face an optimization problem.

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

Fact: You face an optimization problem.

It has a **feasible set**: The set of all valid choices.

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

Fact: You face an optimization problem.

It has a **feasible set**: The set of all valid choices.

It has an **objective function**: The function we are optimizing over the feasible set.

$$f : \left\{ \begin{array}{c} \text{feasible} \\ \text{set} \end{array} \right\} \rightarrow \left\{ \begin{array}{c} \text{some measure} \\ \text{of goodness} \end{array} \right\}$$

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

Fact: You face an optimization problem.

It has a **feasible set**: The set of all valid choices.

It has an **objective function**: The function we are optimizing over the feasible set.

$$f : \left\{ \begin{array}{c} \text{feasible} \\ \text{set} \end{array} \right\} \rightarrow \left\{ \begin{array}{c} \text{some measure} \\ \text{of goodness} \end{array} \right\}$$

Our feasible set is _____ and the objective function is _____.

The language of optimization

In our soccer ball example,

- ▶ Our feasible set is the set of positive integers.
- ▶ The objective function is the total yearly cost associated to delivering every k days.

The language of optimization

In our soccer ball example,

- ▶ Our feasible set is the set of positive integers.
- ▶ The objective function is the total yearly cost associated to delivering every k days.

Things you know:

- ▶ Optimize can mean either maximize or minimize.
- ▶ If $f(x)$ is differentiable on a closed interval (feasible set),
Then the maximum and minimum of $f(x)$ both exist,
And they occur at a critical point or at the boundary.