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★ We are trying to find the optimal ordering schedule.

An ordering schedule example

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- There may be other considerations, such as a maximum or minimum shipment...

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Snickers bar	Gourmet chocolate square
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Swedish Fish	Tootsie roll lollypop
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Our feasible set is ________ and the objective function is ________.

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In our soccer ball example,

- Our feasible set is the set of positive integers.
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Things you know:

- Optimize can mean either maximize or minimize.
- If $f(x)$ is differentiable on a closed interval (feasible set), Then the maximum and minimum of $f(x)$ both exist, And they occur at a critical point or at the boundary.