Optimization: Inventory Policy

As the manager of a large retail store, you sell 20 soccer balls a day.

Question: How often, and how many balls should you order from the factory?

Perhaps 20 each day?

Pros: No need to store the balls. Can adapt to market conditions.

Cons: Pay for delivery each day.

Perhaps a year's worth?

Pros: Only pay delivery once!

Cons: Must store the balls.

Suppose the delivery cost is \$100 per shipment.

Suppose the carrying cost is \$0.05 per ball per day.

★ We are trying to find the optimal ordering schedule.

An ordering schedule example

One possible schedule: Order **100** balls every **five** days.

Day	Delivery?	Delivery Cost	Number in Inventory	Carrying Cost
1	✓	\$100	100	\$5
2	×	\$0	80	\$4
3	×	\$0	60	\$3
4	×	\$0	40	\$2
5	×	\$0	20	\$1
6	✓	\$100	100	\$5

Total delivery cost for 5 days: \$100
Total carrying cost for 5 days: \$5+\$4+\$3+\$2+\$1=\$15.
How many deliveries in a year?
Total yearly cost:

An ordering schedule example

In general: Order $20k$ balls every k days.

Total delivery cost for k days: \$100

Total carrying cost for k days: $\$k + \$(k-1) + \cdots + \$2 + \$1 =$

How many deliveries in a year?

Total yearly cost: $C = \frac{365}{k}\left(100 + \frac{k(k+1)}{2}\right)$.

Find the k that minimizes this function.

Solving $\frac{dC}{dk} = 365\left(-\frac{100}{k^2} + \frac{1}{2}\right) = 0$

Gives $k \approx 14.1$. Answer?

- There may be other considerations, such as a maximum or minimum shipment...

The language of optimization

Optimization questions cover a wide variety of situations.

Example. You are given the choice of **one** of the following candies.

Snickers bar	Gourmet chocolate square
Box of Mike & Ikes	Bounty (Coconut+Almond)
Swedish Fish	Tootsie roll lollypop
Kitkat Bar	Three Marshmallow Peeps
Licorice	Peanut M&M's

Fact: You face an optimization problem.

It has a **feasible set**: The set of all valid choices.

It has an **objective function**: The function we are optimizing over the feasible set.

$$f : \left\{ \begin{array}{c} \text{feasible} \\ \text{set} \end{array} \right\} \to \left\{ \begin{array}{c} \text{some measure} \\ \text{of goodness} \end{array} \right\}$$

Our feasible set is ________ and the objective function is ________.

The language of optimization

In our soccer ball example,

- Our feasible set is the set of positive integers.
- The objective function is the total yearly cost associated to delivering every k days.

Things you know:

- Optimize can mean either maximize or minimize.
- If $f(x)$ is differentiable on a closed interval (feasible set), Then the maximum and minimum of $f(x)$ both exist, And they occur at a critical point or at the boundary.