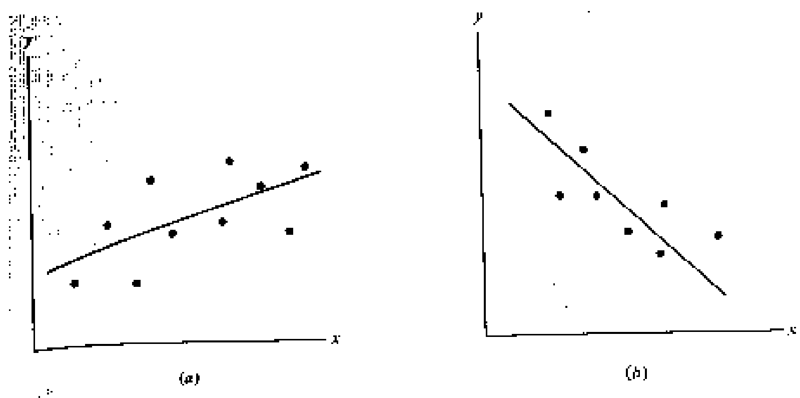
Correlation

Goal: Find cause and effect links between variables.

What can we conclude when two variables are highly **correlated**?



Positive Correlation	**Negative Correlation**
High values of x are associated with high values of y.	High values of x are associated with low values of y.

The **correlation coefficient,** R^2 is a number between 0 and 1.

Values near 1 show strong correlation (data lies almost on a line).
Values near 0 show weak correlation (data doesn't lie on a line).

Calculating the R^2 Statistic

To find R^2, you need data and its best fit *linear* regression. Calculate:

- The **error sum of squares**: $SSE = \sum_i \left[y_i - f(x_i)\right]^2$.

★ SSE is the variation between the data and the function. ★

★ Note: this as what "least squares" minimizes. ★

- The **total corrected sum of squares**: $SST = \sum_i \left[y_i - \bar{y}\right]^2$, where $\bar{y}$ is the average y_i value.

★ SST is the variation solely due to the data. ★

- Now calculate $R^2 = 1 - \frac{SSE}{SST}$.

★ R^2 is the proportion of variation explained by the function. ★

Calculating the R^2 Statistic

Example. (cont'd from notes p. 32) What is R^2 for the data set:
$$\{(1.0, 3.6), (2.1, 2.9), (3.5, 2.2), (4.0, 1.7)\}?$$

You first need the regression line: $f(x) = -0.605027x + 4.20332$.

- The **error sum of squares**: $SSE = \sum_i \left[y_i - f(x_i)\right]^2$.

$$SSE = \big(3.6 - f(1.0)\big)^2 + \big(2.9 - f(2.1)\big)^2 + \big(2.2 - f(3.5)\big)^2 + \big(1.7 - f(4.0)\big)^2$$
$$= (.0017)^2 + (-0.033)^2 + (0.114)^2 + (-0.083)^2 = 0.0210$$

- The **total corrected sum of squares**: $SST = \sum_i \left[y_i - \bar{y}\right]^2$.

First, calculate $\bar{y} = (3.6 + 2.9 + 2.2 + 1.7)/4 = 2.6$

$$SST = (3.6 - 2.6)^2 + (2.9 - 2.6)^2 + (2.2 - 2.6)^2 + (1.7 - 2.6)^2$$
$$= (1)^2 + (0.3)^2 + (-0.4)^2 + (-0.9)^2 = 2.06$$

- Now calculate $R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{0.0210}{2.06} = 1 - .01 = 0.99$.

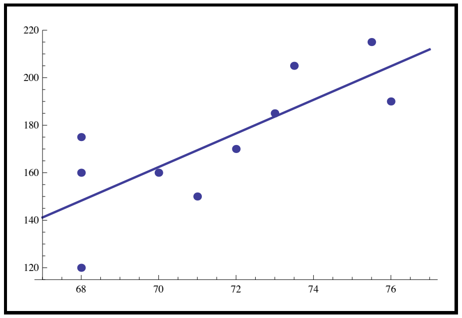
Another R^2 Calculation

Example. Estimating weight from height.

Here is a list of heights and weights for ten students.

We calculate the line of best fit:

$$(\text{weight}) = 7.07(\text{height}) - 333.$$



ht.	wt.
68	160
70	160
71	150
68	120
68	175
76	190
73.5	205
75.5	215
73	185
72	170

Now find the correlation coefficient: $(\overline{w} = 173)$

$SSE = \sum_{i=1}^{10} \left[w_i - (7.07\, h_i - 333)\right]^2 \approx 2808$

$SST = \sum_{i=1}^{10} [w_i - 173]^2 = 6910$

So $R^2 = 1 - (2808/6910) = 0.59$, a good correlation.

We can introduce another variable to see if the fit improves.

Multiple Linear Regression

Add waist measurements to the data!

We wish to calculate a *linear* relationship such as:

$$(\text{weight}) = a\,(\text{height}) + b\,(\text{waist}) + c.$$

Do a regression to find the *best-fit plane*:

Use the least-squares criterion. Minimize:

$$SSE = \sum_{(h_i, ws_i, wt_i)} \left[wt_i - (a \cdot h_i + b \cdot ws_i + c)\right]^2.$$

ht.	wst.	wt.
68	34	160
70	32	160
71	31	150
68	29	120
68	34	175
76	34	190
73.5	38	205
75.5	34	215
73	36	185
72	32	170

This finds that the best fit plane is (coeff sign)

$$(\text{weight}) = 4.59(\text{height}) + 6.35(\text{waist}) - 368.$$

Multiple Linear Regression

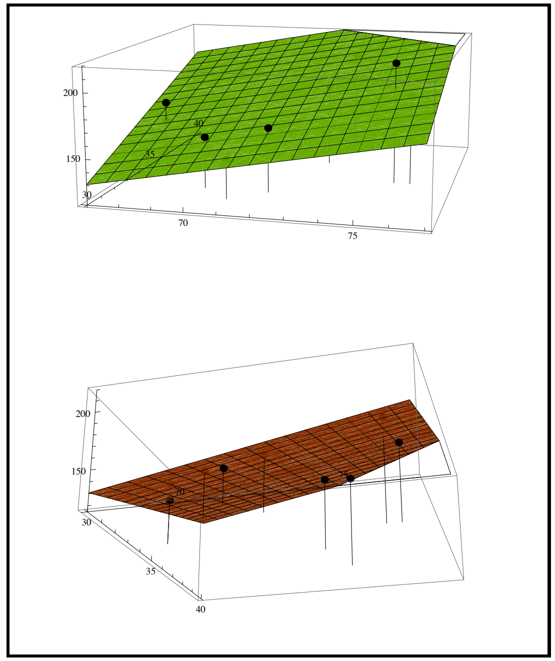
Visually, we might expect a plane to do a better job fitting the points than the line.

- Now calculate R^2.

Calculate $SSE = \sum_{i=1}^{10}(w_i - f(h_i, ws_i))^2 \approx 955$

SST does not change: (why?)

$\sum_{i=1}^{10}(w_i - 173)^2 = 6910$



ht.	wst.	wt.
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68	34	175
76	34	190
73.5	38	205
75.5	34	215
73	36	185
72	32	170

So $R^2 = 1 - (955/6910) = 0.86$, an excellent correlation.

- When you introduce more variables, SSE can only go down, so R^2 always increases.

Notes about the Correlation Coefficient

Example. Time and Distance (pp. 190)
Data collected to predict driving time from home to school.

Variables:
T = driving time S = Last two digits of SSN.
M = miles driven

Use a linear regression to find that
$T = 1.89M + 8.05$, with an $R^2 = 0.867$.

Compare to a multiple linear regression of
$T = 1.7M + 0.0872S + 13.2$, with an $R^2 = 0.883$!

- R^2 increases as the number of variables increase.
- This doesn't mean that the fit is better!

Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

Variables:

T = log of years of fluoridation A = % of population over 65.

C = cancer mortality rate

Use a linear regression to find that
$C = \mathbf{27.1}\,T + 181$, with an $R^2 = 0.047$.

Compare to a multiple linear regression of
$C = \mathbf{0.566}\,T + 10.6A + 85.8$, with an $R^2 = 0.493$.

- Be suspicious of a low R^2.
- Signs of coefficients tell positive/negative correlation.
- Cannot determine relative influence of one variable in one model without some gauge on the magnitude of the data.
- **CAN** determine relative influence of one variable in two models.