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2. **Observation Errors** occur during data collection. **(**$\star$**)**

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3. **Truncation Errors** occur when you approximate an incalculable function.
4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

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1. **Formulation Errors** occur when simplifications or assumptions are made.

Example from the book, pp. 70–73: Seismology.

Set off an explosion at one place and measure it at another (dist. D). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive T, and the second shock T'.

$$d = \frac{D}{2}\sqrt{(T'/T)^2 - 1}$$

Assumptions: The earth is flat, and the layer is parallel to the surface.

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Assumptions: The earth is flat, and the layer is parallel to the surface.

If layers are not parallel (off by α°), the percent errors can be large!

α	1	5	10	30
% error in d	3.4	18	37	105

Errors inherent in the modeling process

2 **Observation Errors** occur during data collection.

Continuation of the previous example:

Even if the layers are parallel, perhaps our timing is inaccurate.
Let's say that T is 1 second and T' is 1.2 seconds,
but that our timer is off by at most 1%.

Then T might be ____ seconds or ____ seconds,
and T' might be ____ seconds or ____ seconds.

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% error in d	-0.5%	-5%	+6%	0%

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One way to decrease influence: measure many times, take average.

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3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

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Question: What is 1.2300001^{10}?

Answer: If we only have three-digit accuracy, then

$1.23 \cdot 1.23 = 1.51, \qquad 1.23 \cdot 1.51 = 1.86 \qquad \ldots \qquad 1.23^{10} = \mathbf{7.95}$

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$1.2300001 \cdot 1.2300001 = 1.5129002,$

$1.2300001 \cdot 1.5129002 = 1.8608674,$

$1.2300001^{10} = \mathbf{7.9259523}$

True answer: $7.92595253991286345258474801873764932003980 5\cdots$

Random Walk

A **random walk** is a sequence of steps, where each step is generated randomly and depends only on its current position.

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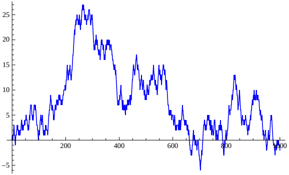
Stock prices



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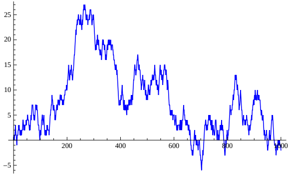
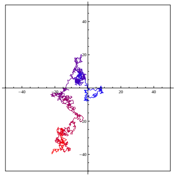
Stock prices **Movement of a molecule in liquid (Wiener process)**





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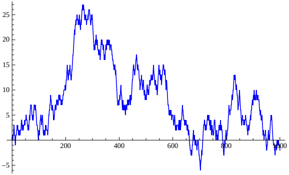

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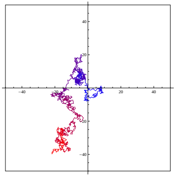

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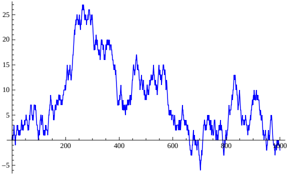

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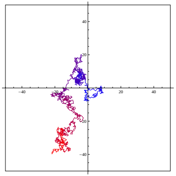

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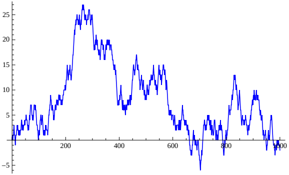

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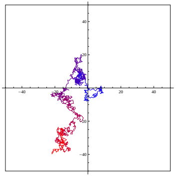

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We transition from one state to another by some rule.Perhaps:

- Moving a random card to a new position.
- Choosing a pair of random cards and exchanging them.

Simple random walk

A drunk in a bar. A bar patron has had a little too much to drink and it's about time to leave the bar. There is an exit directly to his right and an exit three steps away to his left. The drunk stumbles randomly one step to the left or one step to the right with equal probability.

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What is the transition matrix for this random walk?

What is an equilibrium solution for this random walk?

Gambler's Ruin

Win or go home broke! A gambler starts with \$500 and makes \$1 bets, winning each with probability p.
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The probability of winning \$100 before losing \$500 is 0.004486

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There also exist higher-dimensional random walks.

Color mixing game

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- Choose a color. (Red, Orange, Yellow, Green, Blue, Purple)
- Record the distribution.

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  - Randomly decide whose color will prevail.
    (Coin flip or Rock Paper Scissors)
  - Both players now take the winning color.
  - Repeat many times!

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- Record the distribution at multiple times during the experiment.

What do we expect to occur?

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What do we expect to occur?

Stand up and make some space to move around.