Errors inherent in the modeling process

Models always have errors $\rightsquigarrow$

- Be aware of them.
- Understand and account for them!
- <u>Include in model discussion.</u>

Types of Errors

1. **Formulation Errors** occur when simplifications or assumptions are made. **($\star$)**
2. **Observation Errors** occur during data collection. **($\star$)**
3. **Truncation Errors** occur when you approximate an incalculable function.
4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

Errors inherent in the modeling process

1. **Formulation Errors** occur when simplifications or assumptions are made.

Example from the book, pp. 70–73: Seismology.

Set off an explosion at one place and measure it at another (dist. D). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive T, and the second shock T'.

$$d = \frac{D}{2}\sqrt{(T'/T)^2 - 1}$$

Assumptions: The earth is flat, and the layer is parallel to the surface.

If layers are not parallel (off by α°), the percent errors can be large!

α	1	5	10	30
% error in d	3.4	18	37	105

Errors inherent in the modeling process

2 **Observation Errors** occur during data collection.

Continuation of the previous example:

Even if the layers are parallel, perhaps our timing is inaccurate.
Let's say that T is 1 second and T' is 1.2 seconds,
but that our timer is off by at most 1%.

Then T might be ____ seconds or ____ seconds,
and T' might be ____ seconds or ____ seconds.

T	over	over	under	under
T'	over	under	over	under
% error in d	-0.5%	-5%	+6%	0%

One way to decrease influence: measure many times, take average.

Errors inherent in the modeling process

3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

Question: What is 1.2300001^{10}?

Answer: If we only have three-digit accuracy, then

$1.23 \cdot 1.23 = 1.51, \qquad 1.23 \cdot 1.51 = 1.86 \qquad \ldots \qquad 1.23^{10} = \mathbf{7.95}$

$1.2300001 \cdot 1.2300001 = 1.5129002,$

$1.2300001 \cdot 1.5129002 = 1.8608674,$

$1.2300001^{10} = \mathbf{7.9259523}$

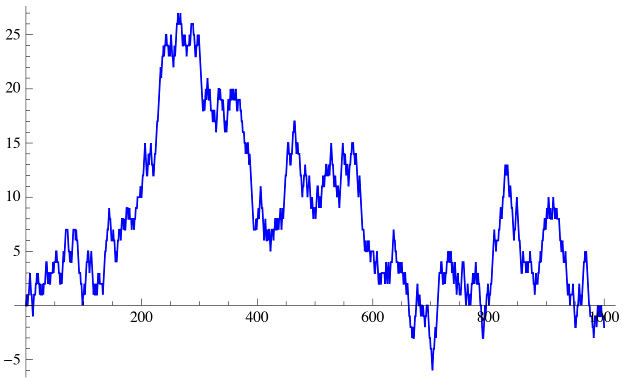
True answer: $7.9259525399128634525847480187376493200398 05 \cdots$

Random Walk

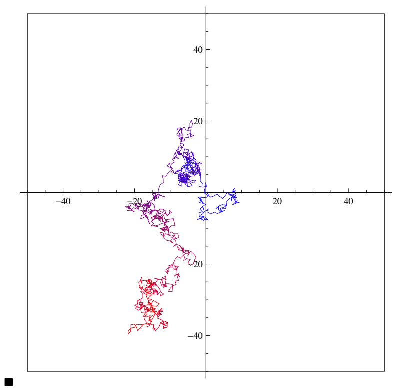
A **random walk** is a sequence of steps, where each step is generated randomly and depends only on its current position.

Random walks can be thought of as a special type of Markov chain.

Stock prices



Movement of a molecule in liquid (Wiener process)



Genetic drift

Diffusion of populations

Polymers

Shuffling of a deck of cards.
Each state is one of the $n!$ permutations of the n cards.
We transition from one state to another by some rule. Perhaps:

- Moving a random card to a new position.
- Choosing a pair of random cards and exchanging them.

Simple random walk

A drunk in a bar. A bar patron has had a little too much to drink and it's about time to leave the bar. There is an exit directly to his right and an exit three steps away to his left. The drunk stumbles randomly one step to the left or one step to the right with equal probability.

What is the probability that the drunk leaves via the right door?

What is the transition matrix for this random walk?

Gambler's Ruin

Win or go home broke! A gambler starts with \$500 and makes \$1 bets, winning each with probability p.
The gambler stays until she has made \$100 profit or goes broke.

Question. What is the probability that she goes home a winner?

This depends on p. For roulette: $p = 18/37 \approx 48.6\%$:

We will model this with a random walk.

Color mixing game

Let's play an interactive Markov chain game.

- Choose a color. (Red, Orange, Yellow, Green, Blue, Purple)
- Record the distribution.
- Do some Markov mixing.
  - Find a random partner. Announce your colors.
  - Randomly decide whose color will prevail.
    (Coin flip or Rock Paper Scissors)
  - Both players now take the winning color.
  - Repeat many times!
- Record the distribution at multiple times during the experiment.

What do we expect to occur?

Stand up and make some space to move around.