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- You can make objective judgements that computers cannot; you know which data points should be taken more seriously.
- Mathematics give precise answers; every answer is fallible.

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One or the other might make more sense depending on the situation.

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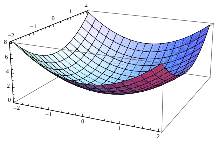

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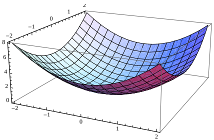

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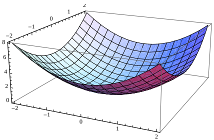

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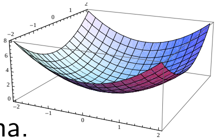
- Solve the resulting system of equations.
- Check to see if the solutions are local minima.



Least Squares Example

Example. Use the least-squares criterion to fit a line $y = mx + b$ to the data: $\{(1.0, 3.6), (2.1, 2.9), (3.5, 2.2), (4.0, 1.7)\}$.

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That is, the line that gives the least-squares fit for the data is

$$y = -0.605027x + 4.20332.$$

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- We'll learn how to use *Mathematica* to do this for us!

Price – Demand Curve (p. 111–114)

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price p	\$9	\$10	\$11
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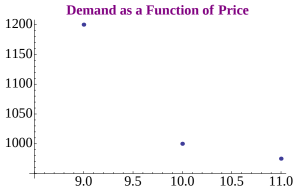
$\rightarrow$ Use the method of least squares to determine this constant c.



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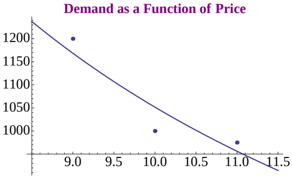
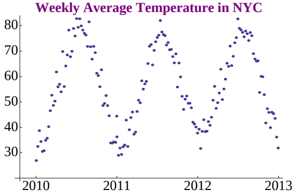
Solving for c gives $c \approx 10517$.



New York City Temperature (similar to p. 158)

The graph of average weekly temperature in New York City from Jan. 2010 to Dec. 2012 gives the distinct impression of a ________.



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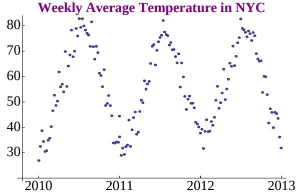

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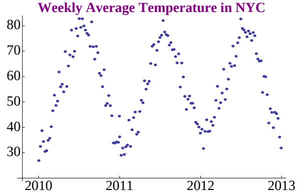

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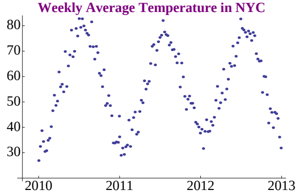

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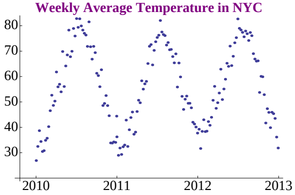

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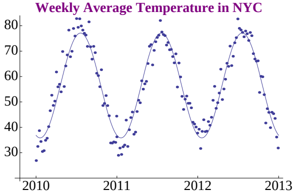

Fitting to $Temp(t) = A + B\sin[2\pi(t - 0.3)]$ gives: $Temp(t) = 56.5 + 20.6\sin[2\pi(t - 0.3)]$