Example. Vehicular Stopping Distance

Background: In driver’s training, you learn a rule for how far behind other cars you are supposed to stay.

▶ Stay back one car length for every 10 mph of speed.
▶ Use the two-second rule: stay two seconds behind.

This is an easy-to-follow rule; it is a safe rule?
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Formulation.
State the question. Identify factors. Describe mathematically.
Culminates with a mathematical model.

Mathematical Manipulation.
Determine mathematical conclusions.

Evaluation.
Translate into real-world conclusions. How good is the model?
Formulation

First, we need to **state the question** (or questions) clearly and precisely.

- Is the two-second rule the same as the 10 mph rule?
- ★ Does the two-second rule mean we’ll stop in time?
- ★★ Determine the total stopping distance of a car as a function of its speed.
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- \( v \) velocity
- \( t_r \) reaction time
- \( a \) vehicle acceleration / deceleration
Breaking down the problem

Describe mathematically.

**Subproblem 1:**
Determine reaction distance $d_r$

**Subproblem 2:**
Determine stopping distance $d_b$
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Determine reaction distance $d_r$
Assume speed is constant throughout reaction distance.
Total reaction distance is
$$d_r = t_r \cdot v.$$  

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Assume brakes applied constantly throughout stopping, producing a constant deceleration.
Brake force is $F = ma$, applied over a breaking distance $d_b$. 
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Brake force is $F = ma$, applied over a breaking distance $d_b$.

This energy absorbs the kinetic energy of the car, $\frac{1}{2}mv$.

Solving $m \cdot a \cdot d_b = \frac{1}{2}mv^2$, we expect

$$d_b = C \cdot v^2.$$
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Therefore, the total stopping distance is $d_r + d_b = t_r \cdot v + Cv^2$. 
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- Did we answer the question?
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Data is available from US Bureau of Public Roads.

Reaction distance is tabulated in Table 2.4 and shown in Figure 2.14. The data lie perfectly (!) on a line. $d_r \approx 1.1v$. 
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▶ Examine methodology of data collection.
▶ Experimenters said \( t_r = 3/4 \) sec and calculated \( d_r \)!
▶ Perhaps we should design our own trial?
Function Fitting

- Compiled data is a range.
  - Trials ran until had a large enough sample
  - Then middle 85% of the trials given.
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► Consider the line in Figure 2.15: $d_b \approx 0.054v^2$.
► Up to 60 mph ($v^2 = 3600$), seems like reasonable fit.
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We conclude that the total stopping distance is

\[
d_{tot} = d_r + d_b \approx 1.1v + 0.054v^2.
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Check fit: Plots observed stopping distance versus model. (Fig. 3.16)

- Model seems reasonable (through 70 mph).
- Residual plot shows additional behavior unmodeled (Fig. 3.17)
How good is the model?

Is the model accurate?

Is the model precise?

Is the model descriptively realistic?

Is the model robust?
How good is the model?

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▶ Suppose that error in is $-10\%$ for a true speed of $v = 60$. 
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Is the model robust?
▶ Suppose that error in is $-10\%$ for a true speed of $v = 60$. Then $v' = 54$ and the model predicts that stopping distance is $1.1 \cdot 54 + 0.054 \cdot 54^2 \approx 217$ instead of $1.1 \cdot 60 + 0.054 \cdot 60^2 \approx 260$. 
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Limitations and assumptions inherent in our model:

Is the model general? When is it reasonable? What are its limitations?

Is the model fruitful? Does it inspire other models? Can it be widely implemented?
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- Drivers going $\leq 70$ mph
- Good road conditions
- Applies when driving cars, not trucks.

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- This line of reasoning can be applied to any situation with constant deceleration.
- Come up with a good rule of thumb for drivers to follow and publicize it. (Next slide!)
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Does the two-second rule mean that we’ll stop in time?

▸ Recognize that a two-second rule is **Easy to implement**.
Vehicular Stopping Distance

Does the two-second rule mean that we’ll stop in time?

- Recognize that a two-second rule is Easy to implement.
- The two-second rule is a linear rule,
- A quadratic rule would make more sense.
- Works up until 40 mph, then quickly invalid! (Figure 2.17)
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Come up with a variable rule based on speed.

▸ It’s not reasonable to tell people to stay 2.5 seconds behind at 50 mph and 2.8 seconds behind at 58 mph!
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► It’s not reasonable to tell people to stay 2.5 seconds behind at 50 mph and 2.8 seconds behind at 58 mph!
► Determine speed ranges where
  ► two seconds is enough
  ► three seconds enough
  ► four seconds enough
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  ▶ two seconds is enough (≤ 40 mph)
  ▶ three seconds enough (≤ 60 mph)
  ▶ four seconds enough (≤ 75 mph)
  ▶ Add more if non-ideal road conditions.