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  - You can determine the amount in the account after one year.

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Probabilistic: Element of chance is involved

- You know the likelihood that something will happen, but you don't know when it will happen.
- *Example.* Roll a die until it comes up '5'.
  - Know that in each roll, a '5' will come up with probability $1/6$.
  - Don't know exactly when, but we can predict well.

Basic Probability

Definition: An **experiment** is any process whose outcome is uncertain.

Definition: The set of all possible outcomes of an experiment is called the **sample space**, denoted X or S.

Definition: Each outcome $x \in X$ has a number between 0 and 1 that measures its likelihood of occurring. This is the **probability** of x, denoted $p(x)$.

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Definition: An **event** E is something that happens (in other words, a subset of the sample space).

Definition: Given E, the **probability** of the event ($p(E)$) is the sum of the probabilities of the outcomes making up the event.

Example. The roll of the die ... [is '5'] or [is odd] or [is prime] ...

Example. $p(E_1) = \underline{\hspace{2cm}}$, $p(E_2) = \underline{\hspace{2cm}}$, $p(E_3) = \underline{\hspace{2cm}}$.

Determining Probabilities

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- **Relative frequency method:**
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- **Equal probability method:** Assume all outcomes have equal probability; assign as the probability $\frac{1}{\#\text{ of possible outcomes}}$.
  Example. Each side of a dodecahedral die is equally likely to appear; decide to set $p(1) = \frac{1}{12}$.
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- **Subjective guess method:** If neither method above applies, give it your best guess.
  Example. How likely is it that your friend will come to a party?

Independent Events

Definition: Two events are **independent** if the probabilities of occurrence do not depend on one another.

Example. Roll a red die and a blue die.

- Event 1: blue die rolls a 1. Event 2: red die rolls a 6. These events are independent.
- Event 1: blue die rolls a 1. Event 2: blue die rolls a 6. These events are dependent.

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Example. Pick a card, any card! Shuffle a deck of 52 cards.

- Event 1: Pick a first card. Event 2: Pick a second card. These events are ____________.

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Example. You wake up and don't know what day it is.

- Event 1: Today is a weekday. E_1 vs. E_2
- Event 2: Today is cloudy. E_2 vs. E_3
- Event 3: Today is Modeling day. E_1 vs. E_3

Independent Events

- When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,

$$p(E_1 \text{ and } E_2) = p(E_1)p(E_2).$$

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Example. What is the probability that you roll a blue 1 OR a red 6?

This does not work with *dependent* events.

Decision Trees

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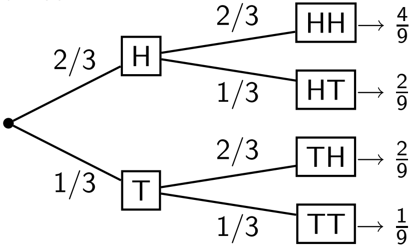

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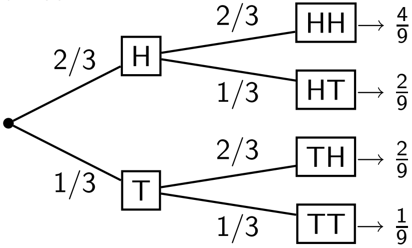

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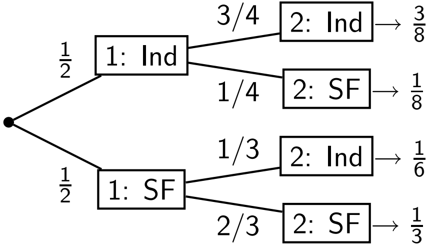
Example. Indiana and SF State U. play two soccer games. (p. 382)



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Definition: The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$\mu = \mathbb{E}[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).$$

Idea: With probability $p(x_1)$, there is a contribution of $r(x_1)$, etc.

Example. How many heads would you expect on average when flipping a biased coin twice?

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Example. How many heads would you expect on average when flipping a biased coin twice?

Example. How many wins do you expect Indiana to have?

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When two random variables are on two **independent** experiments, the expected value operation behaves nicely:

$$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y] \quad \text{and} \quad \mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y].$$

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3	4	5	6	7	8	9
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Component Reliability

Many systems consist of components pieced together. To determine how reliable the **system** is, determine how reliable **each component** is and apply probability rules.

Definition: The **reliability** of a system is its probability of success.

Example. Launch the space shuttle into space with a three-stage rocket.

$$\boxed{\text{Stage 1}} \to \boxed{\text{Stage 2}} \to \boxed{\text{Stage 3}}$$

⋆ In order for the rocket to launch, ______________________ ⋆

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Let $R_1 = 90\%$, $R_2 = 95\%$, $R_3 = 96\%$ be the reliabilities of Stages 1–3.

$p(\text{system success}) = p(\text{S1 success and S2 success and S3 success})$

Component Reliability

Example. Communicating with the space shuttle.
There are two independent methods in which earth can communicate with the space shuttle

- A microwave radio with reliability $R_1 = 0.95$
- An FM radio, with reliability $R_2 = 0.96$.

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$p(\text{system success}) = p(\text{MW radio success or FM radio success})$