Deterministic versus Probabilistic

Deterministic: All data is known beforehand

▶ Once you start the system, you know exactly what is going to happen.
▶ *Example.* Predicting the amount of money in a bank account.
  ▶ If you know the initial deposit, and the interest rate, then:
  ▶ You can determine the amount in the account after one year.

Probabilistic: Element of chance is involved

▶ You know the likelihood that something will happen, but you don't know when it will happen.
▶ *Example.* Roll a die until it comes up '5'.
  ▶ Know that in each roll, a '5' will come up with probability $1/6$.
  ▶ Don't know exactly when, but we can predict well.

Basic Probability

Definition: An **experiment** is any process whose outcome is uncertain.

Definition: The set of all possible outcomes of an experiment is called the **sample space**, denoted X or S.

Definition: Each outcome $x \in X$ has a number between 0 and 1 that measures its likelihood of occurring. This is the **probability** of x, denoted $p(x)$.

Example. Rolling a die is an experiment; the sample space is {_____}. The individual probabilities are all $p(i) = $ ____.

Definition: An **event** E is something that happens (in other words, a subset of the sample space).

Definition: Given E, the **probability** of the event ($p(E)$) is the sum of the probabilities of the outcomes making up the event.

Example. The roll of the die ... [is '5'] or [is odd] or [is prime] ...

Example. $p(E_1) = $_____, $p(E_2) = $_____, $p(E_3) = $_____.

Determining Probabilities

Three methods for determining the probability of an occurrence:

- **Relative frequency method:** Repeat an experiment many times; assign as the probability the fraction $\frac{\text{occurrences}}{\#\text{ experiments run}}$.

  Example. Hit a bulls-eye 17 times out of 100; set the probability of hitting a bulls-eye to be $p(\text{bulls-eye}) = 0.17$.

- **Equal probability method:** Assume all outcomes have equal probability; assign as the probability $\frac{1}{\#\text{ of possible outcomes}}$.

  Example. Each side of a dodecahedral die is equally likely to appear; decide to set $p(1) = \frac{1}{12}$.

- **Subjective guess method:** If neither method above applies, give it your best guess.

  Example. How likely is it that your friend will come to a party?

Independent Events

Definition: Two events are **independent** if the probabilities of occurrence do not depend on one another.

Example. Roll a red die and a blue die.

▶ Event 1: blue die rolls a 1. Event 2: red die rolls a 6.
These events are independent.

▶ Event 1: blue die rolls a 1. Event 2: blue die rolls a 6.
These events are dependent.

Example. Pick a card, any card! Shuffle a deck of 52 cards.

▶ Event 1: Pick a first card. Event 2: Pick a second card.
These events are _____.

Example. You wake up and don't know what day it is.

▶ Event 1: Today is a weekday. E_1 vs. E_2
▶ Event 2: Today is cloudy. E_2 vs. E_3
▶ Event 3: Today is Modeling day. E_1 vs. E_3

Independent Events

▶ When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,
$$p(E_1 \text{ and } E_2) = p(E_1)p(E_2).$$

Example. What is the probability that today is a cloudy weekday?

▶ When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,
$$p(E_1 \text{ or } E_2) = 1 - (1 - P(E_1))(1 - P(E_2))$$
$$= P(E_1) + P(E_2) - p(E_1)p(E_2)$$

Proof: Venn diagram / rectangle

*Example.* What is the probability that you roll a blue 1 OR a red 6?
This does not work with *dependent* events.

Decision Trees

Definition: A **multistage** experiment is one in which each stage is a simpler experiment. They can be represented using a **tree diagram**.
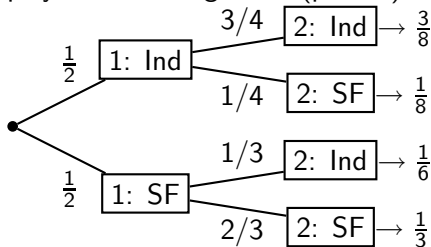
Each branch of the tree represents one outcome x of that level's experiment, and is labeled by $p(x)$.

Example. Flipping a biased coin twice.
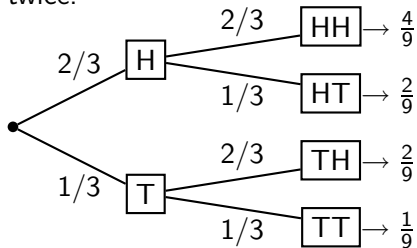
Independent or dependent?

Example. Indiana and SF State U. play two soccer games. (p. 382)

Independent or dependent?

Expected value / mean

"Even with the randomness, what do you expect to happen?"

Suppose that each outcome in a sample space has a number $r(x)$ attached to it. (examples: number of pips on a die, amount of money you win on a bet, inches of precipitation falling)

This function r is called a **random variable**.

Definition: The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$\mu = \mathbb{E}[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).$$

Idea: With probability $p(x_1)$, there is a contribution of $r(x_1)$, etc.

Example. How many heads would you expect on average when flipping a biased coin twice?

Example. How many wins do you expect Indiana to have?

Expected value / mean

When two random variables are on two **independent** experiments, the expected value operation behaves nicely:

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y] \text{ and } \mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y].$$

Example. We throw a red die and a blue die. What is the expected value of the sum of the dice and the product of the dice?

$b+^r$	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

| $b*^r$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$\mathbb{E}[X + Y] =$

$\mathbb{E}[XY] =$

Component Reliability

Many systems consist of components pieced together. To determine how reliable the **system** is, determine how reliable **each component** is and apply probability rules.

Definition: The **reliability** of a system is its probability of success.

Example. Launch the space shuttle into space with a three-stage rocket.

$$\boxed{\text{Stage 1}} \rightarrow \boxed{\text{Stage 2}} \rightarrow \boxed{\text{Stage 3}}$$
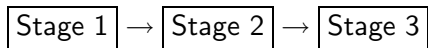
$\star$ In order for the rocket to launch, _____ $\star$

Let $R_1 = 90\%$, $R_2 = 95\%$, $R_3 = 96\%$ be the reliabilities of Stages 1–3.

$p(\text{system success}) = p(\text{S1 success and S2 success and S3 success})$

Component Reliability

Example. Communicating with the space shuttle.
There are two independent methods in which earth can
communicate with the space shuttle

- ▶ A microwave radio with reliability $R_1 = 0.95$
- ▶ An FM radio, with reliability $R_2 = 0.96$.

⋆ In order to be able to communicate with the shuttle,

_____ .

$p(\text{system success}) = p(\text{MW radio success or FM radio success})$