Simulating a doctor's waiting room

Goal: Simulate the queuing behavior at a doctor's office.

Assumptions:

1. Patients arrive on the minutes between 9:00 am and 11:59 am.
2. At most one patient arrives during any minute.
3. The probability that a patient arrives in any one minute is 0.075.
4. Each patient needs to meet with the doctor for 15 minutes.

Expectations:

1. How many patients does the doctor expect to see in the day?

2. How much time does the doctor expect to spend with patients?

Simulating a doctor's waiting room

How do we simulate the arrival of patients?

Assumptions:

1. Patients arrive on the minutes between 9:00 am and 11:59 am.
2. At most one patient arrives during any minute.
3. The probability that a patient arrives in any one minute is 0.075.

We let `i` be the counter for time. `i` = 0 occurs at time 9:00 am.
`i` = ___ occurs at time 11:59 am.

We set up a `For` loop:

```
For[i = 0, i _________, i _____,
   newPatient = If[RandomReal[] <= 0.075, 1, 0] ]
```

Dealing with the waiting

How do we simulate "waiting" in a "waiting room"?

An arriving patient will wait when _____.

We will keep track of the following variables:

$$\texttt{nwait} = \text{The number of patients waiting.}$$

$$\texttt{endTime} = \text{The time when the current patient finishes with the doctor.}$$

$$\texttt{busy} = \begin{cases} 1 \text{ if the doctor is busy} \\ 0 \text{ if the doctor is free} \end{cases}$$

Simulating a doctor's waiting room

Now translate the flowchart into an algorithm:

Pseudocode:

- Zero out the counters.
- For `i` from 0 up to 180,
  - If the doctor is finishing with a patient at time `i`, then set `busy=0`.
  - Determine if a new patient arrives (random, probability 0.075). If so, set `newPatient` to 1; otherwise set `newPatient` to 0.
  - If `newPatient == 1`, add one to the number waiting (`nwait++`)
  - If the doctor is not busy AND there is a patient waiting,
    - Subtract one from the number waiting (`nwait--`)
    - Set the doctor to be busy (`busy = 1`)
    - Set the time when the doctor is not busy (`endTime = i + 15`).

Dealing with the waiting

```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0,
      nwait--; busy = 1; endTime = i + 15];
   ]
```

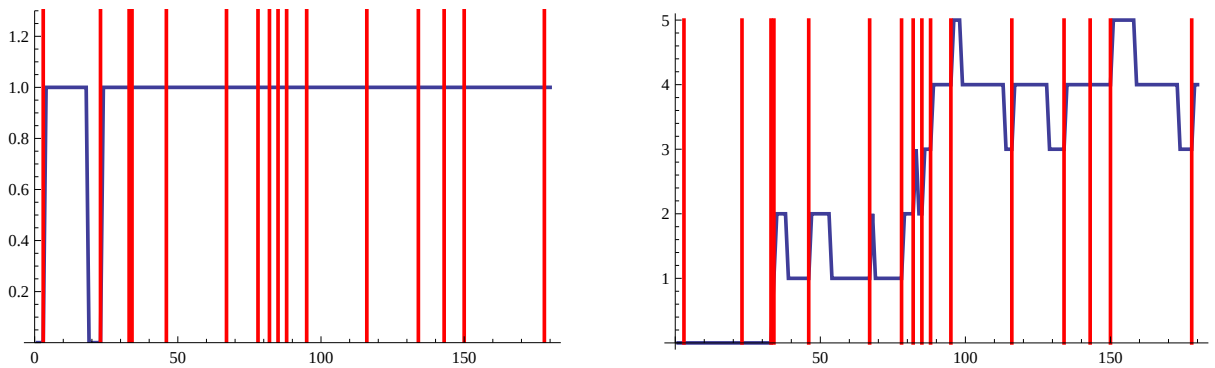
- For i from 0 up to 180,
  - If the doctor is finishing with a patient at time i, then set `busy=0`.
  - Determine if a new patient arrives (random, probability 0.075). If so, set `newPatient` to 1; otherwise set `newPatient` to 0.
  - If `newPatient == 1`, add one to the number waiting (`nwait++`)
  - If the doctor is not busy AND there is a patient waiting,
    - Subtract one from the number waiting (`nwait--`)
    - Set the doctor to be busy (`busy = 1`)
    - Set the time when the doctor is not busy to $i + 15$.

What does the simulation tell us?

We did the simulation, but what was the point?

- How much of the day will the doctor will be busy?
- What is the average number of patients in the waiting room?
- How many people are in the waiting room at noon?
- How late will the doctor stay after noon?

What statistics do we need to keep track of to answer these questions?



This is just one instance; the power of simulation comes from running the model many times and understanding the average behavior.

Gathering data

- How many people are in the waiting room at noon?

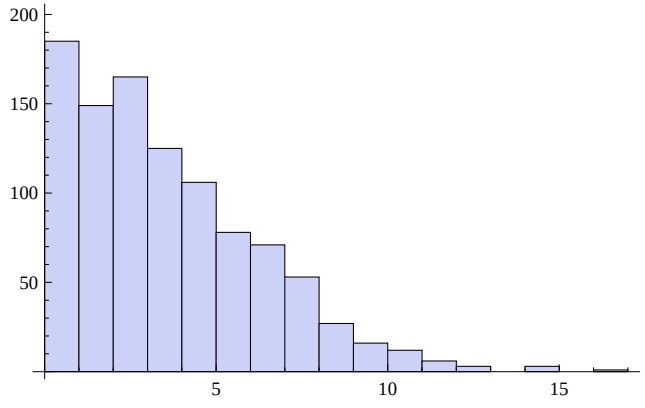
```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0,
      nwait--; busy = 1; endTime = i + 15];
   ];
nwait ⟵ [Outputs the value after the loop ends.]
```

Running many trials

- How many people are in the waiting room at noon?

Simulate 1000 times using a `Table` command, generate a histogram.

```
trials = Table[
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0,
      nwait--; busy = 1; endTime = i + 15];
   ];
nwait ], {j,1000}]
Mean[trials] ⟵ [Average: 3.105]
Histogram[trials]
```



Gathering data

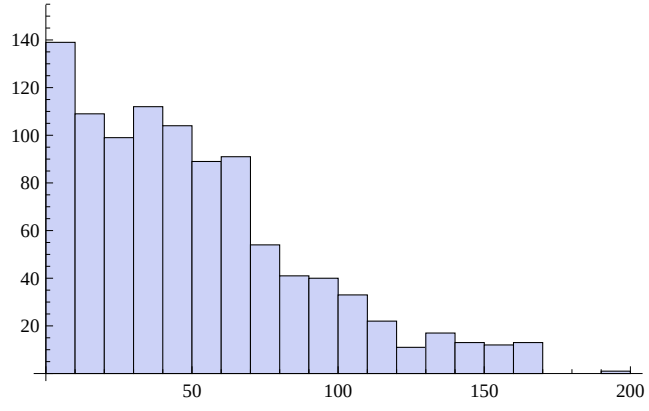
- How late will the doctor stay after noon?

```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0,
      nwait--; busy = 1; endTime = i + 15];
   ];
```

*****

↑ [*How to calculate how long the doctor has to stay after noon?*]

Mean: 51 minutes (does this make sense?)

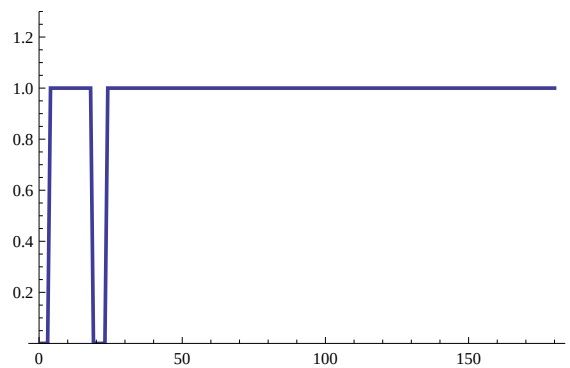


Gathering and plotting time-dependent data

- How much of the day will the doctor will be busy?

Keep track of if the doctor is busy by using a variable `isBusy`.

```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
    If[endTime == i, busy = 0];
    newPatient = If[RandomReal[] <= 0.075, 1, 0];
    If[newPatient == 1, nwait++];
    If[busy == 0 && nwait > 0,
       nwait--; busy = 1; endTime = i + 15];
    isBusy[i]=busy; ⟵ [this copies busy into isBusy[i].]
    ];          ↓ [Puts data into a list.]
busyList=Table[isBusy[i],i,0,179]
Total[busyList] ⟵ [Total time busy.]
ListLinePlot[busyList]
```



Gathering and plotting time-dependent data

- What is the average number of patients in the waiting room?

Keep track of number of waiting patients by using a variable `numWait`.

```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0,
      nwait--; busy = 1; endTime = i + 15];
   numWait[i]=nwait; ⟵ [this copies nwait into numWait[i].]
   ];          ↓ [Puts data into a list.]
waitList=Table[numWait[i],i,0,179]
m=Mean[waitList] ⟵ [Average patients.]
p1=ListLinePlot[waitList];
p2=Plot[m,x,0,179];
Show[p1,p2]
```