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Examples.

- `If[x<0, -x, x]` is the absolute value function. Why?
- `If[RandomInteger[] == 1, "Head", "Tail"]` gives "Head" half the time and gives "Tail" half the time.

Using `If` statements in `Table` commands

Goal: Model a 7.5% chance of occurrence.

Recall that `RandomReal[]` outputs a random number between 0 and 1.

To model a 7.5% chance of occurrence, split the interval at _____.

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Alternatively, do this is one step:

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That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

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trials=Table[If[RandomReal[] <= 0.075, 1, 0],{500}];
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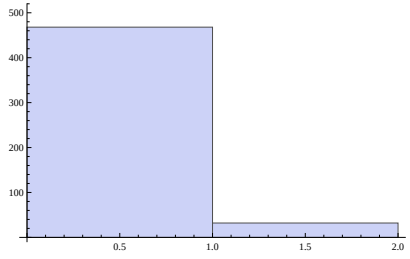
- Last, we might want a visualization;
  Use `Histogram[trials]` to get:



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- Now `i` $= 1$, which is still < 4. So '`Print[i]`' is evaluated and `i` is incremented. Similarly for $i = 2$ and $i = 3$. Now i is incremented to 4, which is NOT < 4, and the loop terminates.

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This variable `i` is called a **counter**.
Be careful to name counters wisely! They are defined as variables.

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Example. Simulate flipping a fair coin 20 times using a `for` loop.

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For[loopCount = 1, loopCount <= 20, loopCount++,
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- Notice the == and also the ; that separates the commands.
- `loopCount` is ONLY a counter; it does not change each step's evaluation.

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```
headCount=0; tailCount=0;
For[loopCount = 1, loopCount <= 20, loopCount++,
  If[RandomInteger[]==1,
      Print["Head"]; headCount++,          ← Notice the ';'
      Print["Tail"]; tailCount++]]         ← Notice the '++'
{headCount, tailCount}
```

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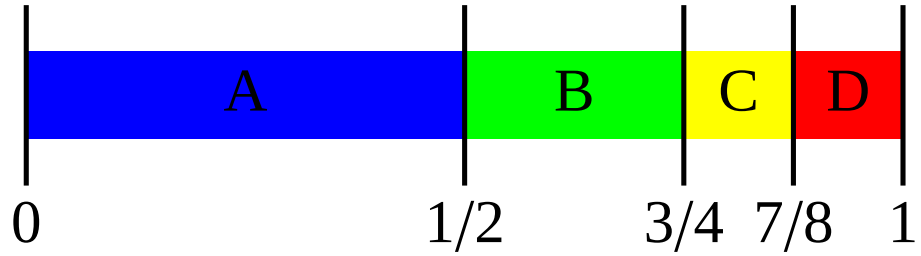

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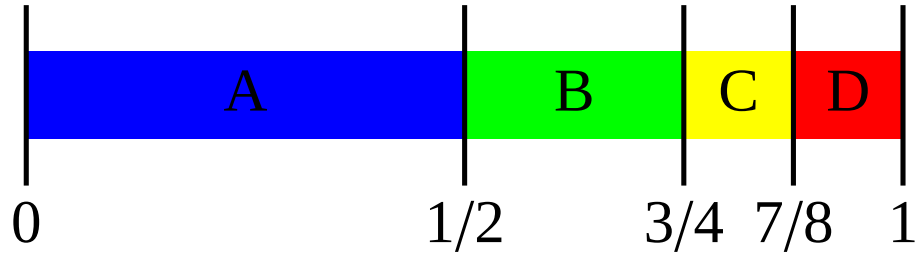

- Reset the counters: 'aCount=bCount=cCount=dCount=0'.
- For loopCount from 1 to 20,
  - Generate a random real number between 0 and 1.
  - If between 0 and 1/2, then output 'A' and aCount++
    if between 1/2 and 3/4, then output 'B' and bCount++
    if between 3/4 and 7/8, then output 'C' and cCount++
    if between 7/8 and 1, then output 'D' and dCount++
- Display 'aCount', 'bCount', 'cCount', and 'dCount'.

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- Sample output: (each on its own line)

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- *Important:* You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.
- If you are feeling fancy, you can use one `Which` command instead of four `If` commands.

Using Simulation to Calculate Area

Suppose you have a region whose area you don't know. You can approximate the area using a Monte Carlo simulation.

Idea: Surround the region by a rectangle. Randomly chosen points in the rectangle will fall in the region with probability

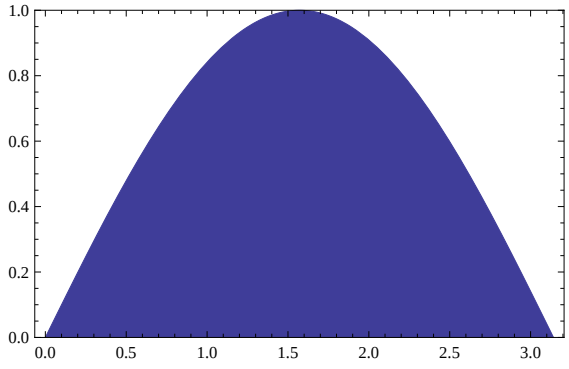
$$\text{(area of region)/(area of rectangle)}$$

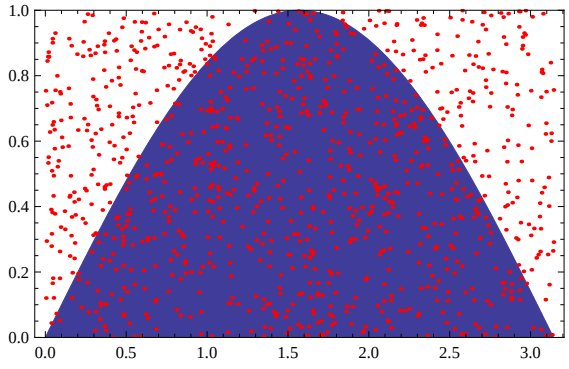
We can approximate this probability by calculating

$$\text{(points falling in region)/(total points chosen)}.$$

Using Simulation to Calculate Area

Example. What is the area under the curve $\sin(x)$ from 0 to π?





Randomly select 100 points from the rectangle $[0, \pi] \times [0, 1]$.

[*Choose a random real between* 0 *and* π *for the* x*-coordinate and a random real between* 0 *and* 1 *for the* y*-coordinate...*]

$$\text{Then, } \frac{\text{Area of region}}{\qquad} \approx \frac{\text{Number of points in region}}{100}.$$

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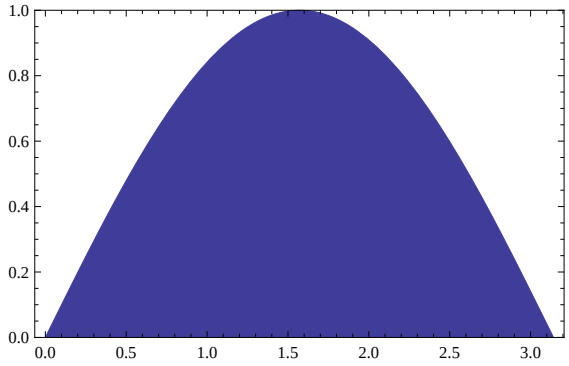

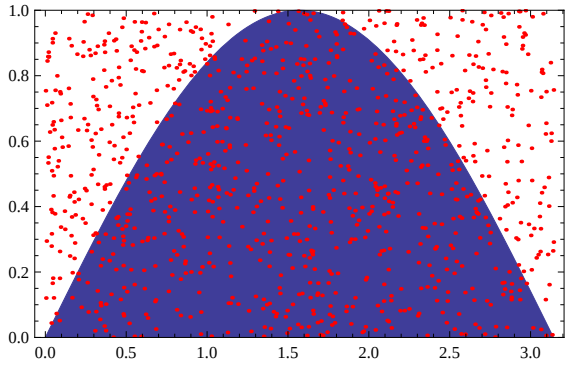

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Here, 63 points fell in the region; we estimate the area to be $\underline{\qquad\qquad}$.

Compare this to the actual value, $\int_{x=0}^{x=\pi} \sin(x)\,dx = [-\cos(x)]_{x=0}^{x=\pi} = 2$