## Resource Allocation (p. 254)

Determine the linear program corresponding to the following problem.

A farmer has set aside 18 acres of land to be used entirely for plots
of grapes, potatoes, and lettuce. Each crop has its own requirements
for labor and capital and a different profit per acre. The farmer has
$540 of capital and 48 hours of labor. How large should each plot be?

| Plot Type | Hours per acre | Capital per acre | Net Profit per acre |
|-----------|----------------|------------------|---------------------|
| Grapes    | 9              | 54               | 60                  |
| Potatoes  | 9              | 26               | 45                  |
| Lettuce   | 3              | 27               | 30                  |

## Resource Allocation (p. 254)

▶ What are the decision variables?
Write out in words what each variable represents.

▶ What is the objective function? What are we trying to
maximize as a function of the decision variables?

▶ What are the constraints? What equalities, inequalities must
be satisfied by the decision variables? (corresponding to what?)

## Diet Problem (p. 255)

Determine the linear program corresponding to the following problem.

The nutrition director of a college wishes to blend a soup to serve the students from two pre-made stocks. Available are an onion stock with 5 units of protein and 10 units of iron at 3¢/ounce and a chicken stock with 7 units of protein and 4 units of iron at 2¢/ounce.

It is determined that the soup must have at least 35 units of protein and 40 units of iron. How many ounces of each should be included to meet or exceed the requirements at least cost?

## Diet Problem (p. 255)

Sketch the feasible region for this problem.

Determine the corner points of the region.

Notice that the feasible region is unbounded. This means there might not be a solution to the problem. (Why?)

Why should we expect that there actually is a solution to this problem?

▶ Real-world justification:

▶ Explanation by simply looking at the linear program:

Determine new prices for the unmixed stocks that would change the optimal solution. [*Hint: the feasible region does not change.*]
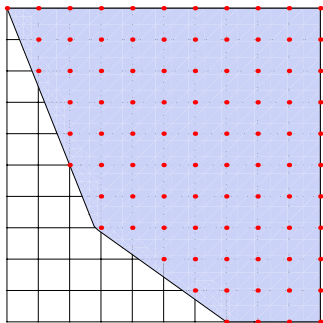
# Integer Programming

It might make sense to restrict the decision variables to be integers. Perhaps it is impossible to measure out fractions of ounces of soup?

*Definition:* A linear program with the added restriction that the decision variables must be integers is called an **integer program**.

One approach to solving an integer program:

▶ Solve the linear program by ignoring the integrality constraints

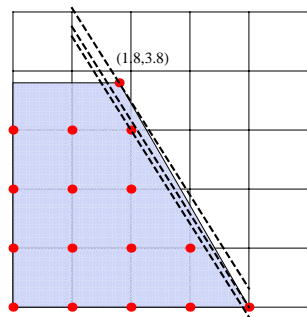▶ Take the solution and round the real-valued solution to the nearest *feasible* integer coordinates.

## Integer Programming

However, problems may arise if we use this method. (p. 276)

▶ It may be difficult to find the nearest feasible point.

▶ Nearest point might not be optimal!

Maximize $3.8x + 2.4y$
  subject to:    $3.8x + 2.2 \leq 15.2$
                        $y \leq 3.8$
                      $x, y \geq 0$

Solving integer programs is **much harder** than solving linear programs.

We still are able to use *Mathematica*:

```
In[1]: Maximize[{3.8x+2.4y, x>=0 && 0<=y<=3.8 &&
                     3.8x+2.2y <= 15.2}, {x, y}, Integers]
Out[1]: {15.2, {x -> 4, y -> 0}}
```

## Application: 0-1 variables

*Example.* Knapsack Problem (p. 277)

A total of $m$ items with fixed weights $a_1$, $a_2$, ..., $a_m$ are to be packed in a knapsack. The total weight cannot exceed a fixed weight, $b$. The objective is to pack as many items as possible.

*Formulation.* Create one decision variable $x_i$ for each item $i$, where we designate that

$$x_i = \begin{cases} 1 & \text{if item } i \text{ is to be packed.} \\ 0 & \text{if item } i \text{ is not to be packed.} \end{cases}.$$

*Now:* What are we trying to maximize? What are our constraints? IP

## Application: 0-1 variables

*Example.* Assignment Problem (p. 258)

Suppose 3 people $P_1, P_2, P_3$ are being considered for 3 jobs, $J_1, J_2, J_3$.

Also suppose that the company knows how much
money each person will make the company daily
completing each job. (Define it to be $a_{ij}$.)

|       | $P_1$ | $P_2$ | $P_3$ |
|-------|-------|-------|-------|
| $J_1$ | 15    | 13    | 12    |
| $J_2$ | 11    | 3     | 9     |
| $J_3$ | 10    | 5     | 7     |

How to assign people to jobs in order to make the most profit?

*Formulation.* Create a 0-1 decision variables $x_{ij}$ to represent:

$$x_{ij} = \begin{cases} 1 & \text{if person } P_i \text{ is given job } J_j. \\ 0 & \text{if person } P_i \text{ is not given job } J_j. \end{cases}.$$

*Goal:* Maximize $\sum_{i,j} a_{ij} x_{ij}$ subject to $x_{ij} = 0$ or 1 and
Job constraints: $x_{11} + x_{12} + x_{13} = 1$, $x_{21} + x_{22} + x_{23} = 1$, $x_{31} + x_{32} + x_{33} = 1$.
Ppl constraints: $x_{11} + x_{21} + x_{31} = 1$, $x_{12} + x_{22} + x_{32} = 1$, $x_{13} + x_{23} + x_{33} = 1$.

# Carpenter's Problem

*Example.* Suppose that a carpenter makes tables and bookcases.
- ▶ Tables require 20 units of lumber and 5 hours of labor.
- ▶ Bookcases require 30 units of lumber and 4 hours of labor.
- ▶ She makes a profit of \$25 per table and \$30 per bookcase.
- ▶ She has on hand 690 units of lumber and 120 units of labor.

Determine the optimal number of tables and bookcases to build.

*Formulation.* Let $x$ be the number of tables and $y$ be the number of bookcases she builds in a week. We have the following LP.

$$\text{maximize} \quad z = 25x + 30y \quad \text{(objective function)}$$

$$\begin{aligned}
\text{subject to} \quad & 20x + 30y \leq 690 && \text{(lumber constraint)} \\
& 5x + 4y \leq 120 && \text{(labor constraint)} \\
& x, y \geq 0 && \text{(nonnegativity constraints)}
\end{aligned}$$

—Worksheet—

## Carpenter's Problem

The worksheet helps us to understand the idea of **sensitivity analysis**.

Given a linear program and its solution,

▶ How sensitive is the solution to changes in the objective function or the constraints?

The economic interpretation of the results is that the equilibrium cost of lumber is about 71.4 cents per unit and that of labor is $2.14 per hour.

▶ So if we can buy our lumber for less or hire labor cheaper than that, we should (up to a point)

▶ And, if we can sell our lumber for more or contract out labor for more, we should (up to a point)

This "point" occurs when the constraint becomes redundant.