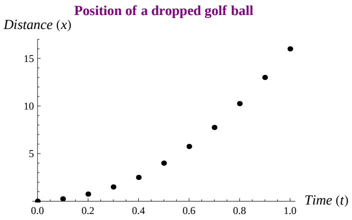
Data Fitting

Suppose you have a set of data, perhaps in scatterplot form.



We wish to help determine function(s) that represent the relationship between the variables involved.

This is called **fitting the data** to a function.

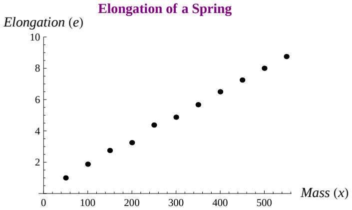
Today we focus on fitting data *visually* (without computations).

Springs and Elongations

Example: Modeling Spring Elongation

Take your favorite spring. Attach different masses. How much does it stretch from rest? [Its **elongation**.]

When we plot the data, we get the following **scatterplot**.



m	e
50	1.000
100	1.875
150	2.750
200	3.250
250	4.375
300	4.875
350	5.675
400	6.500
450	7.250
500	8.000
550	8.750

What do you notice? ____________________

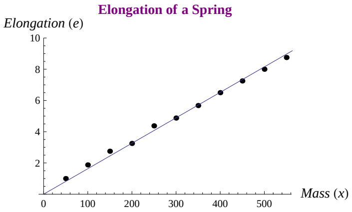
Proportionality

When data seems to lie on a line through the origin, we expect the two variables to be **proportional**; in this case, $e = km$ for some constant k.

We need to find this **constant of proportionality** k.

So: Estimate the slope of the line. **How?**

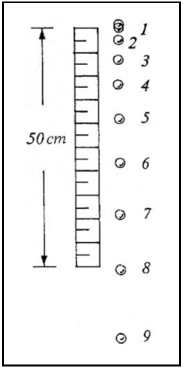
1. Guesstimating



2. Mathematically: **Linear Regression / Least Squares**
(For another day)

Fitting Gravity Data

Example. Modeling the dropping of a golf ball

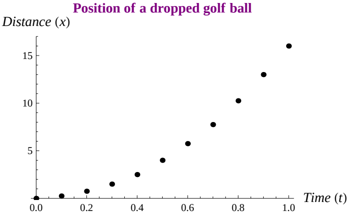


Source:
practicalphysics.org

Let's use an experiment to test the gravity model from last time.

Use a camera to record the position every tenth of a second.

Data would be similar to the table→
It's plotted in the scatterplot below.



t	x
0.0	0.00
0.1	0.25
0.2	0.75
0.3	1.50
0.4	2.50
0.5	4.00
0.6	5.75
0.7	7.75
0.8	10.25
0.9	13.00
1.0	16.00

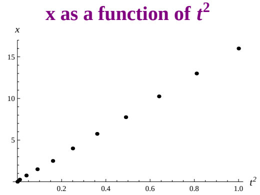
[Ignore data on p. 25.]
[It's a typo.]

Fitting Gravity Data

These data seem to fit a $\underset{\text{(type of function)}}{\underline{\hspace{3cm}}}$. How can we be sure?

1. Plot distance as a function of t^2 and estimate constant of proportionality.

t	t^2	x
0.0		0.00
0.1		0.25
0.2		0.75
0.3		1.50
0.4		2.50
0.5		4.00
0.6		5.75
0.7		7.75
0.8		10.25
0.9		13.00
1.0		16.00



Fitting Gravity Data

When fitting data to a polynomial, an alternate method is:

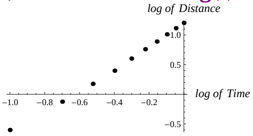
2 ⋆ Plot the log of distance as a function of log of time. ⋆

- WHY? Suppose $x = Ct^k$. Taking a logarithm of both sides,
  $\ln x = \ln(Ct^k) =$

Conclusion: To approximate C and k,

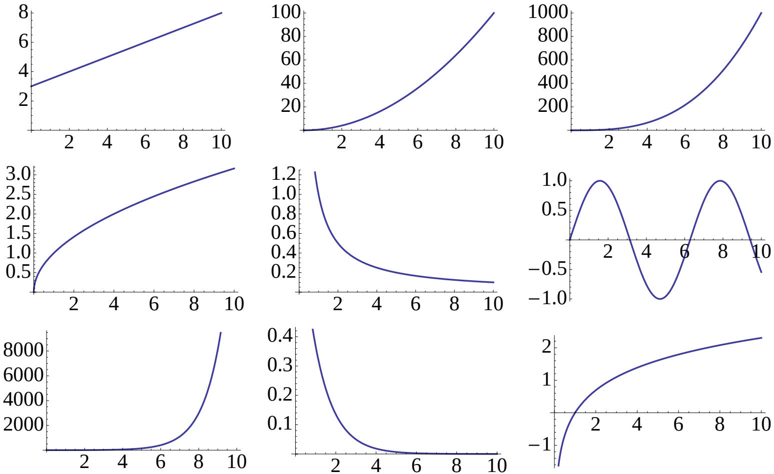
- First, calculate $\ln x$ and $\ln t$ for each datapoint.
- Fit the transformed data to a line.
  - The slope is an approximation for k
  - The y-intercept approximates $\ln C$.

log(x) as a function of log(t)



$\log x \approx 2 \log t + 1.2$

Functions You Should Recognize on Sight

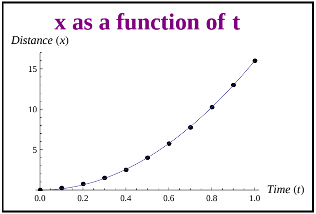


Fitting Gravity Data

We have determined that our gravity model

$$x(t) = 16t^2$$

appears to model the dropping of a golf ball.



Example. Raindrops—Our model gives their position as $x(t) = 16t^2$.

A raindrop falling from 1024 feet would land after $t = 8$ seconds.

However, an experiment shows that the fastest drop takes 40 seconds, and that drops fall at different rates depending on their size.

Even if we have a good model for one situation doesn't mean it will apply everywhere. *We always need to question our assumptions.*

—Extensive gravity discussion in Section 1.3.—