Here are some double angle formulas for you. Enjoy.

$\sin(2\theta) = 2\sin\theta\cos\theta$ • $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ • $\sin^2\theta = \frac{1}{2}\big(1 - \cos(2\theta)\big)$ • $\cos^2\theta = \frac{1}{2}\big(1 + \cos(2\theta)\big)$

1. (5 pts) Give the precise definition of a *smooth curve*.

2. (8 pts = 2 pts each) Determine whether the following statements are **True** or **False**.
   [*No explanation for your answer is required.*]

   (a) **T** or **F**: Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors. Then the quantity $\mathbf{a} \cdot \mathbf{b}$ is always defined and the result is a scalar.

   (b) **T** or **F**: Given two vectors $\mathbf{v} = (v_1, v_2, v_3)$ and $\mathbf{w} = (w_1, w_2, w_3)$, then $\mathbf{v} \times \mathbf{w} = -\mathbf{w} \times \mathbf{v}$.

   (c) **T** or **F**: When defined, the projection of a vector $\mathbf{b}$ onto a vector $\mathbf{a}$ is always parallel to the vector $\mathbf{a}$.

   (d) **T** or **F**: All two dimensional cross sections of a hyperbolic paraboloid (also known as a *saddle*) are hyperbolas.

3. (10 pts) Draw the graphs of each of the following two equations. Explain why you drew what you drew.

   (a) The parametric equations $x(t) = 3t - 4$ and $y(t) = 2t$, for $-1 \le t \le 2$

   (b) The polar equation $\theta = -\pi/6$

4. (12 pts) Find the area inside all leaves of the rose defined by the polar equation

$$r = \sin(6\theta) \text{ for } 0 \le \theta \le 2\pi.$$

   [*Give an exact answer, not a decimal.*]

5. (10 pts) Find the unit tangent vector $\mathbf{T}$ to the curve $\mathbf{r}(t) = \langle \ln t, 2\sqrt{t}, t^2 \rangle$ at the point $(0, 2, 1)$.

6. (10 pts) Set up **but do not evaluate** the integral that finds the arc length of the vector function

$$\mathbf{r}(t) = \big\langle \ln(t), \sin\big(\cos t\big) \big\rangle$$

   from $t = 2$ to $t = 5$.
   [*You* **must** *calculate any derivatives, but there is no need to simplify.*]

7. (10 pts) If a particle is moving on the surface of a sphere of constant radius, show that the position vector and the velocity vector are perpendicular.
   [*Hint: What equation does* $\mathbf{r}(t)$ *satisfy?*]