# Generalizations of Polar Coordinates

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A point can have coords $(r, \theta, z)$:

$(r, \theta)$ are polar coords of $xy$-plane
($r$ is the distance from the $z$-axis)
and $z$ is the "distance" to $xy$-plane

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Useful when problems involve *symmetry about a point*
Spheres, Cones with curved bases.

# Converting from cartesian coordinates

## Cylindrical coordinates

**Conversion:**

$x = r\cos\theta \quad y = r\sin\theta \quad z = z$

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**Conversion:**

$x = \rho\sin\phi\cos\theta$

$z = \rho\cos\phi \quad y = \rho\sin\phi\,\sin\theta$

$\rho^2 = x^2 + y^2 + z^2; \quad \tan\phi = \frac{\sqrt{x^2+y^2}}{z}$

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# Practice

## Cylindrical coordinates

Practice changing coordinates:

$(r, \theta, z) = (2, \frac{2\pi}{3}, 1)$; $(x, y, z) = (3, -3, 7)$

Identify cyl. coord. equations:

**2.** $r = 2$; $z = r^2$; $r^2 - 2z^2 = 4$

**3.** Sketch $r^2 \leq z \leq 2 - r^2$

Convert to cylindrical coordinates

**4.** $x^2 + y^2 + z^2 = 2$; $x^2 + y^2 = 2y$

**5.** Give solid between $x^2 + y^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

**6.** $\left\{ \begin{array}{c} -2 \leq x \leq 2 \\ -\sqrt{4 - x^2} \leq y \leq \sqrt{4 - x^2} \\ \sqrt{x^2 + y^2} \leq z \leq 2 \end{array} \right\}$

## Spherical coordinates

Practice changing coordinates:

$(\rho, \theta, \phi) = (2, \frac{\pi}{4}, \frac{\pi}{3})$;$(x, y, z) = (-1, 1, \sqrt{6})$

Identify sph. coord. equations:

**2.** $\phi = \frac{\pi}{3}$; $\rho \sin\phi = 2$; $\rho = 2\cos\phi$

**3.** Sketch $\left(2 \leq \rho \leq 3 \ \& \ \frac{\pi}{2} \leq \phi \leq \pi\right)$
Sketch $\left(0 \leq \phi \leq \frac{\pi}{3} \ \& \ \rho \leq 2\right)$

Convert to spherical coordinates

**4.** $z = x^2 + y^2$; $z = x^2 - y^2$

**5.** Give solid inside $x^2 + y^2 + z^2 = 4$, above $xy$-plane, below $z = \sqrt{x^2 + y^2}$.

**6.** $\left\{ \begin{array}{c} 0 \leq x \leq 1 \\ 0 \leq y \leq \sqrt{1 - x^2} \\ \sqrt{x^2 + y^2} \leq z \leq \sqrt{2 - x^2 - y^2} \end{array} \right\}$

# Course Evaluation

Please comment on:

1. Prof. Chris's effectiveness as a teacher.
2. Prof. Chris's contribution to your learning.
3. The course material: What you enjoyed and/or found challenging.
4. Is there anything you would change about the course?
5. In the last third of the class, I changed the lecture portion of the class to include electronic slides. In what ways did this change the class? Please provide pros and cons.
6. The assigned Webwork and homework assignments.
7. Is there anything else Prof. Chris should know?

Place completed evaluations in the provided folder.

I will be in my office, Kissena Hall, Room 355.