Fractals and Chaos Simplified for the Life Sciences

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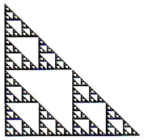
Abbreviated Contents



FRACTALS

Introduction

The Difference between Non-Fractal and Fractal Objects

1. Non-Fractal

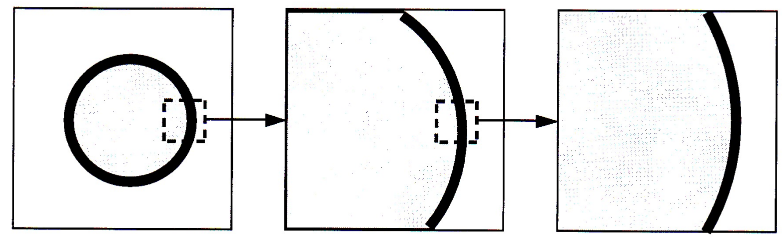
As a **non-fractal** object is magnified, no new features are revealed.

2. Fractal

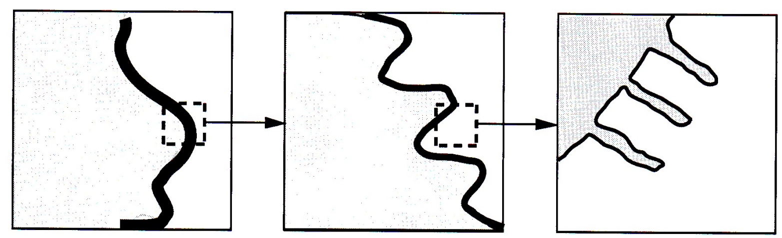
As a **fractal** object is magnified, ever finer features are revealed.

The shapes of the smaller features are kind of like the shapes of the larger features.

Non-Fractal



Fractal



The Sizes of the Features of Non-Fractal and Fractal Objects

1. Non-Fractal

The size of the smallest feature of a **non-fractal** object is called its *characteristic scale.*

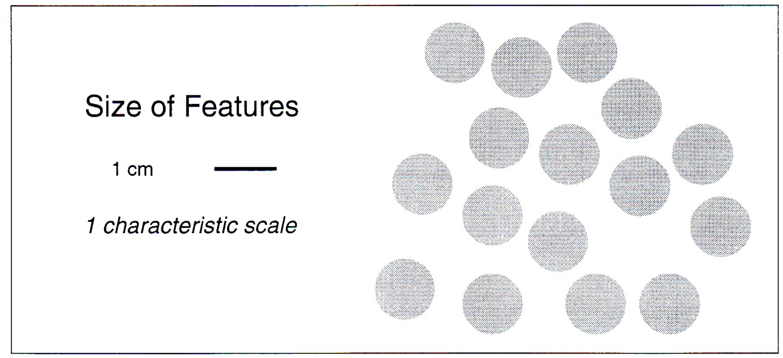
When we measure the length, area, and volume at a resolution that is finer than the characteristic scale, then all of the features of the object are included. Thus the measurements at this resolution determine the correct values of the length, area, and volume.

2. Fractal

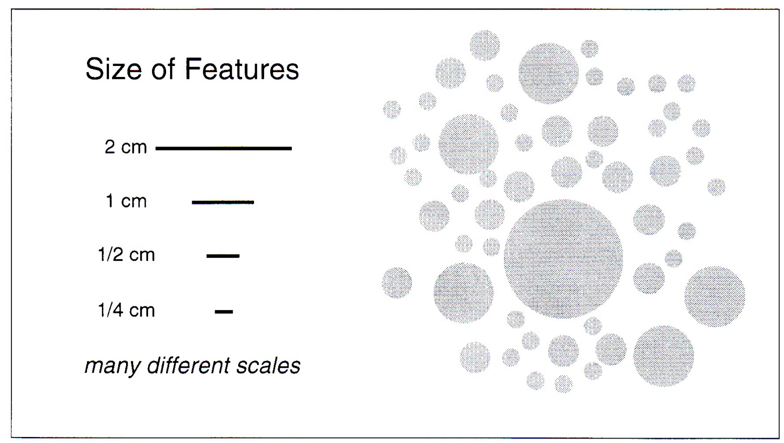
A **fractal** object has features over a broad range of sizes.

As we measure the length, area, and volume at ever finer resolution, we include ever more of its finer features. Thus the length, area, and volume depend on the resolution used to make the measurement.

Non-Fractal



Fractal



The Properties of Fractals

1. Self-Similarity

A coastline looks wiggly. You would think that as you enlarge a piece of the coastline the wiggles would be resolved and the coastline would look smooth. But it doesn't. No matter how much you enlarge the coastline it still looks just as wiggly. The coastline is similar to itself at different magnifications. This is called **self-similarity**.

2. Scaling

Because of self-similarity, features at one spatial resolution are related to features at other spatial resolutions. The smaller features are smaller copies of the larger features. The length measured at finer resolution will be longer because it includes these finer features. How the measured properties depend on the resolution used to make the measurement is called the **scaling relationship**.

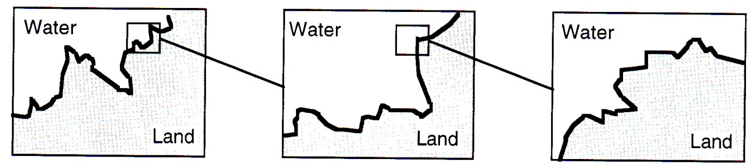
3. Dimension

The dimension gives a quantitative measure of self-similarity and scaling. It tells us how many new pieces of an object are revealed as it is viewed at higher magnification.

4. Statistical Properties

Most likely, the statistics that you were taught in school was limited to the statistics of non-fractal objects. Fractals have different statistical properties that may surprise you!

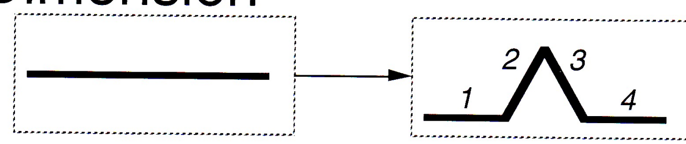
Self-Similarity



Scaling

The value measured for a property depends on the resolution at which it is measured.

Dimension



Statistical Properties

moments may be *<u>zero</u>* *or* *<u>non-finite</u>.*

for example, *mean* $\rightarrow 0$

variance $\rightarrow \infty$